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RATIOS OF INTERPOLATING BLASCHKE PRODUCTS

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Every unimodular function on the unit circle can be uniformly approximated by ratios of interpolating Blaschke products. As a consequence, we show that points of the maximal ideal space of H^∞ can be separated by interpolating Blaschke products.

1. Introduction. Let Δ denote the open unit disc in $\boldsymbol{C}$ and let $H^\infty(\Delta)$ be the Banach algebra of functions bounded and analytic on Δ. A sequence of points $\{z_j\}$ in Δ is called an interpolating sequence if for every bounded sequence $\{\alpha_j\}$ of complex numbers there is a function $F \in H^\infty(\Delta)$ such that

$$F(z_j) = \alpha_j , \qquad j = 1, 2, \cdots .$$

Lennart Carleson [1] has shown that $\{z_j\}$ is an interpolating sequence if and only if

$$\inf_j \prod_{\substack{k \\ k\neq j}} \rho(z_j, z_k) > 0 .$$

Here ρ denotes the pseudo hyperbolic metric; $\rho(w, z) = |(w-z)/(1-\bar{w}z)|$ for $w, z \in \Delta$.

For an arc I on the unit circle $\boldsymbol{T}$ let $|I|$ denote the length of I and let $S(I)$ denote the shadow region of I, $S(I) = \{z \in \Delta\colon (z/|z|) \in I, 1 - |I| < |z| < 1\}$. A positive measure μ on Δ is called a Carleson measure if

$$\sup_I \frac{1}{|I|}\mu(S(I)) = \|\mu\|_C < \infty ,$$

where the above supremum is taken over all arcs I of $\boldsymbol{T}$. There is also a characterization of interpolating sequences in terms of Carleson measures. A sequence $\{z_j\}$ is an interpolating sequence if and only if

(i) $$\inf_{\substack{j\neq k \\ j,k}} \rho(z_j, z_k) > 0$$

and

(ii) $\sum (1 - |z_j|)\delta_{z_j}$ is a Carleson measure ,

where δ_z denotes the Dirac δ measure at z.

A Blaschke product with simple zeros lying on an interpolating sequence is called an interpolating Blaschke product. The purpose