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SAMPLE FUNCTIONS OF POLYA PROCESSES

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For a nonnegative measurable function f satisfying

$$\int_{-\infty}^{\infty} f(x)dx = 1 ,$$

define

$$r(t) = \int_{-\infty}^{\infty} \min\{f(x), f(x+t)\}dx .$$

Berman proved, extending so-called "Polya characteristic function", that the r is the characteristic function of an absolutely continuous distribution. The positive-definiteness of the r corresponds to a stationary Gaussian process, which is called Polya-Covariance process or simply Polya process.

In this paper, some analytic properties of its sample functions are studied: (1) continuity, (2) differentiability, (3) quadratic variation, and (4) upper and lower class.

1. Introduction.

Berman [2] extended a class of characteristic functions described by Polya [7]: Let $f(x)$ be a nonnegative measurable function satisfying

$$\int_{-\infty}^{\infty} f(x)dx = 1 . \tag{1.1}$$

Put

$$r(t) = \int_{-\infty}^{\infty} \min\{f(x), f(x+t)\}dx . \tag{1.2}$$

The r is a characteristic function, corresponding to an absolutely continuous distribution. Since the r is considered as a covariance function, there corresponds to a stationary Gaussian process X with mean zero and with the covariance function r. This process is often called *Polya Covariance Process* [2] or simply *Polya process* (cf: the review for [2], MR 52 (1976), ♯9345).

The Polya process X has a representation by Cabaña and Wschebor, using the plane Wiener process W:

$$X(t) = \int_{(y>0)}\int I\{f(x+t) - y\} W(dx \times dy) ,$$

where $I(u) = 1$ for $u > 0$ and $I(u) = 0$ for $u \leqq 0$ (Berman [2]). This representation is not used in this paper, but in terms of the covariance