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ON FINITE SUMS OF REGRESSIVE ISOLS

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Our paper deals with isols that can be represented as a finite sum of regressive isols, and with recursive functions of one variable that have canonical extensions that map isols such as these into the isols.

1. Preliminaries. We shall use familiar notation in the theory of isols. We denote by ω, Λ and Λ_R respectively, the sets of non-negative integers (*numbers*), isols, and regressive isols. If f is any recursive function, in any number of variables, then f_Λ denotes the *canonical extension* of f to the isols. The *degree of unsolvability* of a regressive isol is defined in the following way, as it is introduced in [3]. Each regressive isol contains a retraceable set, and all retraceable sets that belong to the same regressive isol will have the same Turing degree of unsolvability. If a is a regressive isol, then its *degree of unsolvability* is defined to be the Turing degree of any retraceable set that is a member of a. For a regressive isol a, Δ_a will denote the degree of unsolvability of a. It is proved in [3], that if a and b are any infinite regressive isols, then

1. $a \leqq b \longrightarrow \Delta_a = \Delta_b$,
2. $a \leqq^* b \longrightarrow \Delta_b \leqq \Delta_a$, and
3. $a + b$ regressive implies $\Delta_a = \Delta_b$.

Several times in the paper we use refinement property of isols. This property refers to the following feature of the isols, and it is obtained in [6, corollary to Theorem 19]. If y and $a_0, \cdots, a_m$ are any isols, and

$$y \leqq a_0 + \cdots + a_m ,$$

then there will be isols $y_0, \cdots, y_m$ with $y = y_0 + \cdots + y_m$ and with each $y_i \leqq a_i$.

2. Finite sums of regressive isols. Let $m \geqq 1$ be any number. We set

$$m\Lambda_R = \{a_0 + \cdots + a_{m-1} \mid a_0, \cdots, a_{m-1} \in \Lambda_R\} .$$

When $m = 1$ then $m\Lambda_R$ is simply the collection of all regressive isols. It is easy to see from their definitions, that one has

$$\omega \subset \Lambda_R \subset 2\Lambda_R \subset 3\Lambda_R \subset \cdots .$$