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AN ACTION OF THE AUTOMORPHISM GROUP OF A COMMUTATIVE RING ON ITS BRAUER GROUP

F. R. DEMEYER

An action of the automorphism group of a commutative ring on its Brauer group is given. The action is characterized cohomologically. Relations with the Teichmuller cocycle map and the Schur subgroup are pointed out.

In [13] G. J. Janusz gave an action of the automorphism group of a field K on its Brauer group $B(K)$. For number fields he characterized this action in terms of Hasse invariants and applied his results to the problem of the existence of an outer automorphism of the rational group algebra of a finite group.

Here we give an action of the automorphism group of a commutative ring R on its Brauer group $B(R)$ and describe the action cohomologically. Let A be an Azumaya R-algebra and let σ be an automorphism of R. Define a new R-algebra ${}_\sigma A$ by letting $A = {}_\sigma A$ as rings and with R-module action given by $r*a = \sigma^{-1}(r)a$ for $r \in R$, $x \in A$ where multiplication on the right is in A. Proposition 2 is the assertion that the correspondence $A \rightarrow {}_\sigma A$ induces an action of the group $\mathrm{Aut}(R)$ of automorphisms of R on $B(R)$.

Let L be a finite Galois field extension of K with finite Galois group G and let $\mathrm{Aut}(K: L)$ be the group of automorphisms of K which can be extended to L. In [11] S. Eilenberg and S. MacLane gave an action of $\mathrm{Aut}(K: L)$ on $H^n(G, L^*)$ for $n \geqq 0$. This action corresponds under the natural identification between $B(L/K)$ and $H^2(G, L^*)$ with Janusz's action on $B(L/K)$. For a commutative ring R, $B(R)$ is given as the torsion subgroup of $H^2_{et}(R, U)$ [12] and $H^2_{et}(R, U)$ is a limit of Amitsur cohomology groups [17]. For a faithfully flat commutative extension S of R we give an action of $\mathrm{Aut}(R: S)$ on $H^n(S/R, U)$ (Amitsur cohomology) and show this action commutes with the natural homomorphism given, for example, in [17] from $B(R)$ into $H^2_{et}(R, U)$. We study the problem of extending an automorphism from R to an R-algebra A and its relation to normal algebras and the Teichmuller cocycle map. We show that if K is a field of characteristic $= 0$ then $\mathrm{Aut}(K)$ must always leave the Schur subgroup of $B(K)$ invariant, and we calculate some examples. Throughout all unexplained terminology and notation will be as in [14]. I would like to thank G. J. Janusz, D. Saltman, and D. Zelinsky for helpful remarks.

1. Let R denote a commutative ring, $\sigma \in \mathrm{Aut}(R)$, and let M be