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A NOTE ON H^1 q-MARTINGALES

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Characterizations of H^1 q-martingales via conjugate transforms are studied. Applications to lacunary Fourier series and to local field analysis are given.

1. Introduction. Characterizations of H^1 space over a local field and H^1 regular martingales by singular integral transforms have been studied in a series of papers ([4], [1], [5], [7], and [2]). That is to say,

$$H^1 = \{f: f,\ T_j f \in L^1,\ j = 1, 2, \cdots, m\}$$

where H^1 is the space of regular functions or martingales with integrable maximal functions and T_j, $j = 1, 2, \cdots, m$, are some sort of nice singular integral transforms. In [4], [1] and [5], multiplier transforms arised from multiplicative characters on local fields are used. In [7], singular integral transforms with matrix operators acting on differences of regular martingales are considered. The dyadic case has been excluded until recently. Two methods of handling the dyadic case are given in [2]. First by noting that the maximal function of a dyadic martingale is equivalent in L^1-norm to the maximal function of its "associated 4-martingale", the space of H^1 dyadic martingales is characterized by 4×4 matrix transforms. Then H^1 space over the dyadic number field is studied via the multiplier transform associated to a multiplicative character of ramification degree 2.

In this note, we shall extend the above concept of higher ramification degree to q-martingales and obtain characterizations for the space H^1. We also show that the conditions are necessary. These results provide an answer to an open problem posed by Gundy and Varopoulos in [6]. Applications to homogeneous and nonhomogeneous multipliers on local fields are also given.

2. Conjugate characterizations. Let q be an integer larger than 1. Let $\{\mathscr{F}_n\}_{n \geq 0}$ be an increasing sequence of σ-fields which are generated by atoms in a probability space such that each atom in $\mathscr{F}_n$ contains exactly q atoms in $\mathscr{F}_{n+1}$ of equal measure. One example having such a structure is the group Z_q^∞. Another is the ring of integers of a local field whose residue class field has q elements. A martingale $f = \{f_n\}$ relative to $\{\mathscr{F}_n\}$ is called a *q-adic martingale* or, simply, a *q-martingale.*