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ON MAPS RELATED TO σ-LOCALLY FINITE AND σ-DISCRETE COLLECTIONS OF SETS

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Over the past decade or so, two closely related classes of maps have been introduced and studied independently: The co-σ-discrete maps of R. W. Hansell and the σ-locally finite maps of the author. The principal purpose of this note is, first, to study the relationship between these maps and introduce consistent terminology (which Hansell is also adopting), and second, to prove a theorem which relates these maps to quotient s-maps between metric spaces.

The paper is arranged as follows. General properties of our maps, which were previously studied by Hansell and the author in [4], [5], [9], [11], are established in §§ 2-4. After summarizing some (mostly known) results relating our maps to open maps and closed maps in § 5, we shall prove a result (see Theorem 6.1) about quotient s-maps between metric spaces which will be applied by Hansell in [6]. In § 7 we obtain some examples related to results in §§ 5 and 6, and in § 8 we extend a result from [11] and fill a gap in its proof.

No continuity or separation properties are assumed unless explicitly indicated.

2. Base-σ-locally finite and base-σ-discrete maps. We begin by establishing some terminology. The collection of subsets of X is denoted by $\mathscr{P}(X)$. If $\mathscr{E}, \mathscr{F} \subset \mathscr{P}(X)$, then $\mathscr{F}$ is a *refinement* of $\mathscr{E}$ if $\bigcup \mathscr{F} = \bigcup \mathscr{E}$ and every $F \in \mathscr{F}$ is a subset of some $E \in \mathscr{E}$, and $\mathscr{F}$ is a *base* for $\mathscr{E}$ if every $E \in \mathscr{E}$ is a union of elements of $\mathscr{F}$.[1] If $\mathscr{E} \subset \mathscr{P}(X)$, then $\mathscr{E}$ is *locally finite* (resp. *discrete*) if every $x \in X$ has a neighborhood intersecting at most finitely many (resp. one) $E \in \mathscr{E}$; we call $\mathscr{E}$ *σ-locally finite* (resp. *σ-discrete*) if $E = \bigcup_{n=1}^{\infty} \mathscr{E}_n$ with each $\mathscr{E}_n$ locally finite (resp. discrete).

DEFINITION 2.1. A map $f: X \to Y$ is *base-σ-locally finite* (resp. *base-σ-discrete*) if, whenever $\mathscr{E} \subset \mathscr{P}(X)$ is locally finite (resp. discrete), then $f(\mathscr{E})$ has a σ-locally finite (resp. σ-discrete) base.[2]

Most of our results for base-σ-locally finite and base-σ-discrete

[1] This definition of a base for $\mathscr{E}$ is due to Hansell [**5**]. A base for $\mathscr{E}$ which is also a refinement for $\mathscr{E}$ was called a *baselike refinement* for $\mathscr{E}$ by the author in [**11**]; clearly every base for $\mathscr{E}$ contains a baselike refinement for $\mathscr{E}$.

[2] Base-σ-discrete maps were called co-σ-discrete by Hansell in [**4**] and [**5**]. (Note, however, that base-σ-locally finite maps do *not* coincide with the σ-locally finite maps introduced in [**11**]; see Footnote 4).