PACIFIC JOURNAL OF MATHEMATICS
Vol. 98, No. 2, 1982

PRIME IDEALS IN GAMMA RINGS

SHOJI KYUNO

The notion of a Γ-ring was first introduced by Nobusawa. The class of Γ-rings contains not only all rings but also Hestenes ternary rings. Recently, the author proved the following two theorems: THEOREM A. Let M be a Γ-ring with right and left unities and R be the right operator ring. Then, the lattice of two-sided ideals of M is isomorphic to the lattice of two-sided ideals of R. THEOREM B. Let M be a Γ-ring such that $x \in M\Gamma x\Gamma M$ for every $x \in M$. If $\mathscr{P}(M)$ is the prime radical of the Γ-ring M, then $\mathscr{P}(M_{m,n}) = (\mathscr{P}(M))_{m,n}$. If a Γ-ring M has no unit elements, Theorem A is not, in general, the case. However, it is possible to establish for any Γ-ring M, with or without right and left unities, the result corresponding to Theorem A for a special type of ideals, namely, prime ideals. In this note, we prove Theorem 1. The set of all prime ideals of a Γ-ring M and the set of all prime ideals of the right (left) operator ring $R(L)$ of M are bijective. Applying this result to the matrix $\Gamma_{n,m}$-ring $M_{m,n}$, we obtain Theorem 2. The prime ideals of the $\Gamma_{n,m}$-ring $M_{m,n}$ are the sets $P_{m,n}$ corresponding to the prime ideals P of the Γ-ring M, and Corollary 2. If $\mathscr{P}(M)$ is the prime radical of the Γ-ring M, then $\mathscr{P}(M_{m,n}) = (\mathscr{P}(M))_{m,n}$. This corollary omits the assumption of Theorem B.

1. Preliminaries. Let M and Γ be additive abelian groups. If for $a, b, c \in M$ and $\gamma, \delta \in \Gamma$ the following conditions are satisfied,

(1) $a\gamma b \in M$,

(2) $(a + b)\gamma c = a\gamma c + b\gamma c$, $a(\gamma + \delta)b = a\gamma b + a\delta b$, $a\gamma(b + c) = a\gamma b + a\gamma c$,

(3) $(a\gamma b)\delta c = a\gamma(b\delta c)$,

then M is called a Γ-ring. If A and B are subsets of a Γ-ring M and $\theta \subseteqq \Gamma$, we denote by $A\Theta B$, the subset of M consisting of all finite sums of the form $\sum_i a_i\gamma_i b_i$, where $a_i \in A$, $b_i \in B$ and $\gamma_i \in \Theta$. A right (left) ideal of a Γ-ring M is an additive subgroup I of M such that $I\Gamma M \subseteqq I(M\Gamma I \subseteqq I)$. If I is both a right and a left ideal, then we say that I is an ideal or a two-sided ideal of M. An ideal P of a Γ-ring M is prime if for any ideals $A, B \subseteqq M$, $A\Gamma B \subseteqq P$ implies $A \subseteqq P$ or $B \subseteqq P$. The prime radical $\mathscr{P}(M)$ is defined to be the intersection of all prime ideals of M.

Let M be a Γ-ring and F be the free abelian group generated by $\Gamma \times M$. Then, $A = \{\sum_i n_i(\gamma_i, x_i) \in F \mid a \in M \Rightarrow \sum_i n_i a\gamma_i x_i = 0\}$ is a subgroup of F. Let $R = F/A$, the factor group, and denote the