PACIFIC JOURNAL OF MATHEMATICS
Vol. 98, No. 2, 1982

ON g-METRIZABILITY

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We show that a regular topological space is g-metrizable if and only if it is weakly first countable and admits a σ-locally finite k-network and that a g-metrizable space need not be g-developable.

0. **Introduction.** G-metrizable spaces were defined in [8], where it was also shown that a space admits a countable weak base if and only if it is weakly first countable and has a countable k-network. In this paper we provide the corresponding result for g-metrizable spaces and give an example of a g-metrizable space which is not g-developable. The former result is in response to a question in [8], the latter answers a question in [6]. All spaces are at least regular.

1. **Definition.**

1.1. Let X be a space. If Γ is a family of subsets of X and $\zeta: \Gamma \to \mathscr{P}(X)$ is a function, then the pair $\langle \Gamma, \zeta \rangle$ is a weak base for X if, in addition, the following hold:

(a) For every member G of Γ, $\zeta(G)$ is a subset of G.

(b) If G_1 and G_2 are members of Γ and x is an element of $\zeta(G_1) \cap \zeta(G_2)$, then there is a member G_3 of Γ so that x is in $\zeta(G_3)$ and G_3 is a subset of $G_1 \cap G_2$.

(c) A subset U of X is open if and only if for every element x of U there is a member G of Γ so that x is in $\zeta(G)$ and U contains. G.

This definition of weak base differs from that of [1], namely, a collection $\mathscr{B} = \cup\{T_x: x \in X\}$ is a weak base for X if a set U is open in X precisely when for each point $x \in U$ there exists $B \in T_x$ such that $B \subset U$. It is easy to see that our definition is equivalent to this, for if B is as above, we let $\Gamma = \mathscr{B}$ and for $G \in \Gamma$, let $\delta(G) = \{x: G \in T_x\}$ and if $\langle \Gamma, \delta \rangle$ is a weak base by 1.1, then we let $T_x = \{G: x \in \delta(G)\}$ and $\mathscr{B} = \cup\{T_x: x \in X\}$.

1.2. A space X is g-metrizable if it has a weak base $\langle \Gamma, \zeta \rangle$ where Γ is a σ-locally finite family. X is weakly first countable if X has a weak base $\langle \Gamma, \zeta \rangle$ so that the family $\{\zeta(G): G \in \Gamma\}$ is point countable or, equivalently, there is a function $B: \omega \times X \to \mathscr{P}(X)$ (called a wfc system for X) so that

(a) for all $n < \omega$ and $x \in X$, $B(n+1, x) \subset B(n, x)$;

(b) for all x in X, $x \in \cap\{B(n, x): n < \omega\}$