PACIPIC JOURNAL OF MATHEMATICS
Vol. 99, No. 1, 1982

PSEUDOCOMPACT AND STONE-WEIERSTRASS PRODUCT SPACES

R. M. STEPHENSON, JR.

In 1960 H. Tamano proved that for pseudocompact completely regular spaces X and Y, (i) $X \times Y$ is pseudocompact if and only if pr_X is z-closed, and (ii) $X \times Y$ is pseudocompact if one of X and Y is a k-space.

In 1979 C. E. Aull asked if every product of functionally regular SW spaces is an SW space, and he proved that for a family of functionally regular SW spaces, (iii) their product is an SW space if and only if it is pseudocompact.

The main results of this paper will answer Aull's question affirmatively and prove that (i), (ii), and (iii) hold for strongly functionally Hausdorff spaces.

A topological space X is said to be *functionally Hausdorff* if $C(X)$ (or $C^*(X)$), the set of (bounded) continuous real valued functions defined on X, is point separating; given a functionally Hausdorff space X, wX will denote the completely regular space which has the same points and continuous real valued functions as those of X. A functionally Hausdorff space X will be called an SW *space* if every point separating subalgebra of $C^*(X)$ which contains the constants is uniformly dense in $C^*(X)$ (or, equivalently, if wX is compact). A Hausdorff space X will be called *functionally regular* (*strongly functionally Hausdorff*) if for each point $p \in X$ and neighborhood V of p (such that $V = \mathrm{Cl}(\mathrm{Int}(V))$) there is a zero set Z of X with $p \in Z \subset V$.

In addition to the above, a proof will be given that every feebly compact product of SW spaces is an SW space (which will partially answer another question of Aull), and an example will be given of a non SW product space each of whose finite subproducts is an SW strongly functionally Hausdorff space. This example will show that there exists a sequence $\{X_n\}$ of strongly functionally Hausdorff spaces whose product $X = \prod\{X_n\}$ and whose finite subproducts $X_F = \prod\{X_n: n \in F\}$, F finite, have the following curious properties: each $wX_F = \prod\{wX_n: n \in F\}$ and is compact, but wX fails to be pseudocompact and thus does not equal $\prod\{wX_n\}$.

We will obtain several of these results by proving the following for a product space $X = \prod\{X_a\}$, where each X_a is a pseudocompact strongly functionally Hausdorff space: If X is pseudocompact, or if each factor space X_a is functionally regular, then $wX = \prod\{wX_a\}$.