PACIFIC JOURNAL OF MATHEMATICS
Vol. 99, No. 2, 1982

TRANSVERSALS TO LAMINATIONS

RUSSELL B. WALKER

The stable and unstable manifolds of an Anosov diffeomorphism are not leaves of C^1-foliation. Instead, their unions comprise two laminations; that is, two C^0-foliations which have C^1-smooth leaves and continuous nonsingular tangent plane fields. Recently C. Ennis has shown that laminations have transversals at every point. In this note, the existence of transversals is shown to require plane field continuity.

For these purposes, a C^0-foliation with C^1-smooth leaves will be called an *erratic lamination*. These may contain infinite sequences of points, $\{p_k\} \to p_0$ having tangent planes which do not limit on the tangent plane through p_0.

The example of Theorem 2 is of a 1-dimensional erratic lamination of $\boldsymbol{R}^2$ containing a leaf having no differentiable transversals. Though higher-dimensional, lower codimentional analogues most certainly do exist, the discussion and definitions to follow will be limitted to 1-dimensional foliations.

A C^0-imbedd $(n-1)$-disk D contained in an n-manifold is *topologically transverse* to the leaf of a C^0-foliation if at each point of their intersection, the leaf crosses the disk in a single point. The terms "strictly ingressing" or "strictly egressing" are used similarly in flow theory [3]. D is *topologically transverse* to a C^0-foliation if it is topologically transverse to every leaf. A C^1-imbedded disk is *differentiably transverse* to an erratic lamination if it is differentiably transverse to every leaf. Erratic laminations are the most general foliations for which differentiably transverse disks may exist. A good reference for further definitions and theorems is B. Lawson's survey article, [5].

The Existence of transversals.

The following two theorems distinguish laminations from erratic laminations by the behavior of their topological transversals.

THEOREM 1 (*Ennis* [2], 1979): *Any C^0-imbedded $(n-1)$-disk, D, topologically transverse to a* 1*-dimensional lamination, $\mathscr{L}$ of M^n, can be C^0-approximated by a C^1-imbedded, differentiably transverse disk.*

THEOREM 2. *There exists a* 1*-dimensional lamination $\mathscr{L}$ of* $\boldsymbol{R}^2$