

CRYSTALLISATION MOVES

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A crystallisation of a closed PL-manifold is an edge-coloured graph, which represents it via a contracted triangulation. Any two crystallisations of the same manifold are proved to be joined by a finite sequence of moves, two alternative sets of which are defined. A further move for dimension 3 is introduced.

1. Introduction and notation. Throughout this work balls, spheres, manifolds and maps are piecewise-linear (in the sense of [9] and [17]).

An n -dimensional ball-complex K will be said to be a *pseudocomplex*¹ if each h -ball, considered with all its faces, is abstractly isomorphic with an h -simplex. Further, K will be said to be a *contracted n -complex* if the number of its vertices is $n + 1$. We shall also call *simplex* each element of a pseudocomplex.

By a *pseudodissection* (resp. *contracted triangulation*) of a polyhedron P we mean a pair (K, f) , where K is a pseudocomplex (resp. a contracted complex) and $f: |K| \rightarrow P$ is a homeomorphism. About pseudodissections, compare also [11].

A theorem of Pezzana states that every closed, connected, n -dimensional manifold admits a contracted triangulation (for proofs see [14] and [6] and the sketch contained in this work, §4). The theorem can be extended to manifolds with connected boundary and to more general spaces (see [2]). A method for constructing pseudocomplexes from a set of disjoint n -simplexes is shown in [6] and [7].

Note that even in a pseudodissected manifold, stars and links of simplexes are not necessarily balls and spheres. But there exists a minimal set of severings on them, which makes them balls and spheres. The so modified stars (resp. links) are called *disjoined stars* (resp. *disjoined links*); compare the quoted papers for detailed definitions and proofs.

Closely related is the notion of $(n + 1)$ -coloured graph. Let $\Gamma = (X, E)$ be a finite nonoriented multigraph without loops, $\mathcal{C}$ a set (called *colour-set*), $\gamma: E \rightarrow \mathcal{C}$ a map (called a *coloration*). Such a pair (Γ, γ) is defined to be an *h -coloured graph with boundary* if:

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¹ In all references pseudocomplexes are symbolised with a tild on top of the letter, which we omit throughout this work.