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LEFT THICK TO LEFT LUMPY—A GUIDED TOUR

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We are concerned with locally compact semitopological semigroups, with the variations for such semigroups of the notions of left amenability and left thickness, and with systematizing the many results which generalize a theorem of T. Mitchell for discrete semigroups: A subset T of S is large enough to support a left-invariant mean on S if and only if T is left thick; that is, for each finite subset F of S there is a v in S such that $\{fv \mid f \in F\}$ is a subset of T.

In Part I: The textures of left thickness, we list many variations of left thickness which have already been used, place them in a pattern of $90 = 5 \times 3 \times 3 \times 2$ such conditions, and show that these fall into not more than six equivalence classes. In Part II: The flavors of left-amenability, we list various kinds of amenability that have already been used and try to match them with appropriate thickness conditions; that is we try to find what thickness a set T in S must have to support a given kind of left amenability, supposing, of course, that S itself supports that much amenability. In this part we are also concerned with the thickness which a subsemigroup S' of S needs in order that some kind of amenability of S' forces the same property on S.

1. Preliminaries. T. Mitchell [6] invented the notion of left thickness for a discrete semigroup S in order to characterize in S itself those subsets T of S large enough to support a mean μ which is left-invariant under S. When the same issue is raised for locally compact semitopological semigroups the simplicity of Mitchell's characterization vanishes into a fog of alternative formulations; many of these [3, 4, 8, 9 and 10], have been chosen to suit one or another form of left-invariance that has been found useful by someone at some time.

Always S is a locally compact (Hausdorff) semitopological semigroup; that is, multiplication is associative and is separately or jointly continuous. M is the space of all regular Borel measures on S and P is the subset of M consisting of all probability measures. P_c is the subset of P consisting of measures with compact support. $\delta(S)$ is the subset of P containing all the evaluation functionals $\{\delta_s \mid s \in S\}$. We say that a probability measure ν is *on* a set T if $\nu(T) = 1$, and that ν is *supported on* T if the support of ν is contained in T.

We alse need to recall from Wong [8] and B. Johnson [5] that even when multiplication $\pi(s, t) = st$ is only separately continuous in