PACIFIC JOURNAL OF MATHEMATICS
Vol. 101, No. 1, 1982

ON THE EVALUATION OF PERMANENTS

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Two identities involving permanents are obtained. One of them is used to deduce in a simple way, two well known formulas for the evaluation of permanents, namely the formulas of Binet and Minc and of Ryser.

1. Notation. Let $A = [a_{ij}]$ be an $n \times n$ matrix. The permanent of A is defined by

$$\operatorname{per}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i\sigma(i)} ,$$

where S_n is the symmetric group of degree n.

Let $\Gamma_{r,n}$ denote the set of all n^r sequences $\omega = (\omega_1, \cdots, \omega_r)$ of integers satisfying $1 \leqq \omega_i \leqq n$. Let $Q_{r,n}$ be the set of those sequences in $\Gamma_{r,n}$ which are strictly increasing.

By $G(n)$ we denote the set of all nondecreasing sequences of positive integers $(t_1, \cdots, t_k)$ such that $t_1 + \cdots + t_k = n$.

Given an $n \times n$ matrix A and nonnegative integers $\alpha_1, \cdots, \alpha_n$ $(\beta_1, \cdots, \beta_n)$ satisfying $\alpha_1 + \cdots + \alpha_n = n$ $(\beta_1 + \cdots + \beta_n = n)$, we represent by $A(\alpha_1, \cdots, \alpha_n)$ $(A(\beta_1, \cdots, \beta_n))$ the matrix obtained from A by repeating its first row (column) $\alpha_1(\beta_1)$ times, its second row (column) $\alpha_2(\beta_2)$ times $\cdots$ and its nth row (column) $\alpha_n(\beta_n)$ times. If $\alpha_i = 0(\beta_i = 0)$ the ith row (column) of A is omitted.

Given an $n \times n$ matrix A and nonnegative integers $\alpha_1, \cdots, \alpha_n$, $\beta_1, \cdots, \beta_n$ satisfying $\alpha_1 + \cdots + \alpha_n = n$, $\beta_1 + \cdots + \beta_n = n$, let $A(\alpha_1, \cdots, \alpha_n; \beta_1, \cdots, \beta_n)$ denote the $n \times n$ matrix obtained from A by repeating its first row α_1 times, $\cdots$, its nth row α_n times and also its first column β_1 times $\cdots$ its nth column β_n times. Again $\alpha_i = 0$ or $\beta_i = 0$ means that the ith row or the ith column of A is omitted.

Given the integers α_i, $1 \leqq i \leqq n$, such that $\sum_{i=1}^{n} \alpha_i = n$, let $R_{\alpha_1, \cdots \alpha_n} = (j_1, \cdots, j_n)$ represent the nondecreasing sequence of nonnegative integers $(j_1, \cdots, j_n)$ where i occurs with multiplicity α_i.

2. Two identities involving permanents. Let A be an $n \times n$ matrix and

$$Z = \prod_{i=1}^{n} Z_i$$

with

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