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INDEPENDENCE OF NORMAL WEIERSTRASS POINTS UNDER DEFORMATION

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Let X denote a compact Riemann surface of genus g and suppose $P \in X$. The Weierstrass nongaps at P are those positive integers n such that there exists a meromorphic function on X which has a pole of order n at P and is holomorphic everywhere else. The Weierstrass semigroup at P, which we denote by $\Gamma(P)$, is the additive semigroup consisting of 0 and the nongaps. A point P is a Weierstrass point if there exists a nongap less than $g+1$ at P and is a normal Weierstrass point if $\Gamma(P)=\{0, g, g+2, g+3, g+4, \cdots\}$. We consider here the following problem: Given a collection $P_1, \cdots, P_n$ of points on X, describe the infinitesimal variations of complex structure on X which preserve the Weierstrass semigroups at $P_1, \cdots, P_n$. Our main result says, roughly speaking, that normal Weierstrass points deform as independently of each other as possible.

1. Let T_g denote the Teichmuller space for Teichmuller surfaces of genus $g > 1$ and let $\pi: V \to T_g$ denote the universal curve of genus g. Let $\mathscr{W}_k^r$, for $k = 2, \cdots, 2g-2$ and $r = 1, 2, \cdots$, denote the closed complex subspaces of V of Weierstrass points of the universal curve which were defined in [1], [2]. These spaces may be described set-theoretically as follows:

1) for $k \leqq g$, then
$|\mathscr{W}_k^r| = \{(t, P) \in V$: in the Weierstrass gap sequence at $P \in V_t$, there are at least r nongaps $\leqq k\}$.

2) For $k \geqq g$, then
$|\mathscr{W}_k^r| = \{(t, P) \in V$: in the Weierstrass gap sequence at $P \in V_t$, there are at least r gaps $> k\}$.

If P is a point on a compact Riemann surface, let $\Gamma(P)$ denote the semigroup of Weierstrass nongaps at P. Let Γ be an additive subsemigroup of the nonnegative integers and suppose Γ has g gaps.

DEFINITION. Put $\mathscr{W}(\Gamma) = \{(t, P) \in V: \Gamma(P) = \Gamma\}$.

It is not hard to see that $\mathscr{W}(\Gamma)$ is a (possibly empty) open subset of a finite intersection of $\mathscr{W}_k^r$'s. Thus $\mathscr{W}(\Gamma)$ has the structure of a complex analytic subspace of V.

Suppose $(t, P) \in V$ and put $X = V_t$. Let $c_1, \cdots, c_{3g-3}$ denote Patt's local coordinates [3] on T_g centered at t. These coordinates arise as variation parameters used in performing local variations of structure around $3g-3$ generally chosen points on X. Let z be a