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A NOTE ON STOCHASTIC METHODS IN CONNECTION WITH APPROXIMATION THEOREMS FOR POSITIVE LINEAR OPERATORS

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In this paper some general approximation theorems in probability theory are used in order to deduce assertions on rates of convergence for several concrete positive linear operators, defined on the space of real continuous functions, towards the identity. Similar results are also established on the approximation of such operators towards an operator connected with the Gaussian distribution.

1. Intoduction. Any problem concerning positive linear operators on $C(J)$, the space of continuous real-valued functions defined on an interval $J \subset \boldsymbol{R}$, can naturally be interpreted as a problem in probability. A real random variable (r.v.) Y or r.v. Y_n are associated to the positive, linear operators L, $L_n: C(J) \to C(J)$, $n \in \boldsymbol{N}$, by setting

$$Lf(t) := E(f \circ Y) \ ; \qquad L_n f(t) := E(f \circ Y_n)$$

for fixed $t \in J$, $E(Y)$ denoting the expectation of Y (see e.g., [9], [10], [11]). Apart from [8] which is restricted to pure convergence assertions, apparently all papers on this subject have in common the fact that the structure of the r.v. Y, Y_n is not described any closer, although in the applications one uses that the r.v. are normalized sums.

The starting point of this paper are sequences of independent, not necessarily identically destributed r.v. $(X_i)_{i \in N}$ (defined on a common probability space $(\Omega, \mathscr{A}, P)$) with distribution $P_{X_i}(B) := P(\{\omega \in \Omega; X_i(\omega) \in B\})$, where B is a Borel set of $\boldsymbol{R}$, variance $\operatorname{Var}(X_i)$ and distribution function (d.f.) $F_{X_i}(x) := P_{X_i}((-\infty, x])$, $x \in \boldsymbol{R}$. The aim is to study the convergence behavior of the normalized sums $\varphi(n)S_n$, where $S_n := \sum_{i=1}^n X_i$, and φ is an arbitrary normalizing function $\varphi: N \to \{x \in \boldsymbol{R}, x > 0\} = R^+ \backslash \{0\}$ with $\varphi(n) = o(1)$, $n \to \infty$, towards different limiting r.v., namely $X_0 := 0$ a.s. and X^* (see page 9 for the definition). Examining the convergence towards the first r.v. X_0 corresponds, from the point of view of approximation theory, to the more important case of convergence of the operators L_n towards I. It will turn out to be rather advantageous, to consider normalized sums $\varphi(n)S_n$ instead of arbitrary r.v. Y_n since, on the one hand, this will not lead to any restrictions in the applications. On the other hand, it is now possible to describe the