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A HAUSDORFF-YOUNG INEQUALITY FOR B-CONVEX BANACH SPACES

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A vector valued analogue of the classical Hausdorff-Young inequalities for characters of groups is obtained.

Introduction. For Banach space notions and terminology not explained here, we refer the reader to [5], [6] and the several papers which are mentioned further on. Let us start by recalling the definition of type and cotype of normed spaces. We say that a normed space $X, \|\ \|$ has type p (resp. cotype q) if there is a constant $M < \infty$ (resp. $\delta > 0$) such that for every integer m and every choice of vectors $(x_i)_{1 \leqq i \leqq m}$ in X

$$\left\{\int \|\sum \varepsilon_i(t)x_i\|^2 dt\right\}^{1/2} \leqq M(\sum \|x_i\|^p)^{1/p} \tag{1}$$

respectively

$$\left\{\int \|\sum \varepsilon_i(t)x_i\|^2 dt\right\}^{1/2} \geqq \delta(\sum \|x_i\|^q)^{1/q} \tag{2}$$

holds, where (ε_i) denotes the sequence of Rademacker functions. Take further p_X the supremum of all types $1 \leqq p \leqq 2$ of X and q_X the infimum of all cotypes $2 \leqq q \leqq \infty$. The space X is said to have type (resp. cotype) provided $p_X > 1$ (resp. $q_X < \infty$).

The numbers p_X and q_X have a geometrical interpretation. As shown in [7], if X is an infinite dimensional Banach space, then ℓ^{p_X} and ℓ^{q_X} are both finitely representable in X (see also [8]). In particular, X has type (resp. cotype) if and only if ℓ^1 (resp. ℓ^∞) is not finitely representable in X. The first of those properties is also called B-convexity, a notion which was introduced in [4]. Very recently, see [12], it was proved that if X is B-convex, then p_X and q_{X^*} are conjugate exponents, i.e., $(p_X)^{-1} + (q_{X^*})^{-1} = 1$.

One may ask for an analogue of (1), (2) if the Rademacker functions (ε_i) are replaced by distinct Walsh functions $w_S = \prod_{i \in S} \varepsilon_i$ on the Cantor group $\{1, -1\}^N$ or, more generally, by characters of an arbitrary compact abelian group (integrating with respect to the Haar measure). In this spirit, we will prove

THEOREM 1. *If X is a B-convex Banach space, there exist some $\tilde{p} > 1$ and $\tilde{q} < \infty$ and constants $M < \infty$, $\delta > 0$, such that*

$$\left\{\int \|\sum x_\gamma \gamma(t)\|^2 dt\right\}^{1/2} \leqq M(\sum \|x_\gamma\|^{\tilde{p}})^{1/\tilde{p}} \tag{3}$$