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AUTOMORPHISMS OF QUOTIENTS OF $\prod GL(n_i)$

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Quotients of $GL(n)$ by finite subgroups can have radial algebraic automorphisms. More generally, quotients of $\Pi_1^s GL(n_i)$ by $(s-1)$-dimensional central subgroups can have automorphisms not induced by automorphisms of $\Pi GL(n_i)$. This paper works out an explicit description of all their algebraic group automorphisms. As a sample application, the normalizer of the $GL(n)$-action on $\Lambda^r(k^n)$ is computed.

The automorphisms of the general linear groups $GL(n, k)$ over a field k are quite well known [2, 4]. There are first of all the algebraic automorphisms, which (for $n > 2$) are just the inner automorphisms and transpose inverse. There are also the automorphisms induced by automorphisms of k. Finally, there may in some cases be radial automorphisms sending g to $\lambda(g)g$ for scalar $\lambda(g)$. Such radial automorphisms exist only when k has special properties; they cannot be defined systematically over rings containing k—that is to say, they are not algebraic automorphisms. Consequently, I was rather surprised when I observed that certain naturally occurring images of $GL(n)$ (quotients by finite central subgroups) do have algebraic radial automorphisms. The existence of such automorphisms seems not to have been pointed out before. It turns out to be implicit in one familiar context, but there the group is in disguise (see § 3).

In this paper we will work out precisely when such radial algebraic automorphisms exist and what they can be. More generally we will treat quotients $(\prod GL(n_i))/A$ that have one-dimensional center, and we will go on to compute the whole group of algebraic automorphisms. This will be interesting because a number of outer automorphisms here require appropriate scalar factors in their definition and are not simply induced by automorphisms of $\prod GL(n_i)$. The exact result also is useful when one wants to find the normalizers of these groups in larger ones, and we will conclude with a detailed example of such an application.

For brevity "group" will mean an algebraic group over a field k, and "homomorphism" will mean an algebraic homomorphism. More precisely, we will treat our objects as affine group schemes [5]. The groups $\prod GL(n_i)/A$ that we really care about will have the same automorphisms in any version of algebraic group theory, since they are smooth (and indeed are determined by their k-rational