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THE EXACT SEQUENCE OF A LOCALIZATION FOR WITT GROUPS II: NUMERICAL INVARIANTS OF ODD-DIMENSIONAL SURGERY OBSTRUCTIONS

WILLIAM PARDON

The propose of this paper is to define numerical invariants of odd-dimensional surgery obstructions, computable in a way similar to that used to compute the index and Arf invariants of even-dimensional surgery obstructions. The main result is that a system of integral congruences ("numerical invariants") suffices, modulo the projective class group, to determine whether or not an odd-dimensional surgery obstruction vanishes, when the fundumental group is a finite 2-group. In addition, the numerical invariants turn out to be Euler characteristics in certain cases of topological interest, including the existence of product formulas.

Let π be a group and $Z\pi$ its integral group ring, with the involution induced by $g \rightarrow g^{-1}$, $g \in \pi$. The even-dimensional surgery obstruction group $L_{2n}(Z\pi)$ is, roughly speaking, the Grothendieck group on isometry classes of hermitian forms over $Z\pi$, modulo the subgroup generated by hyperbolic forms. A striking fact, discovered by C. T. C. Wall ([56, §6]), is that the odd-dimensional surgery obstruction group. $L_{2n+1}(Z\pi)$, is (again roughly) the commutator quotient of the group of isometries of the stable hyperbolic form. An important consequence of this result is that the obvious analogy between L_{2n} and L_{2n+1} on the one hand, and K_0 and K_1 on the other, can be used to translate techniques from algebraic K-theory to unitary K-theory. This has been done by many authors.

In spite of this conceptual connection between L_{2n} and L_{2n+1}, however, there remains an important difference between them. Classical invariants of quadratic forms, such as the index or Arf invariant, have been easier to compute than any known algebraic invariants of the unitary group; and, on the geometric side, the braid diagram (in [56, §6]) necessary to construct the odd-dimensional obstruction seems to contain more delicate geometric information than the intersection and self-intersection forms of the even-dimensional case. The purpose of this paper is to define algebraic invariants of odd-dimensional surgery, by a procedure analogous to the one furnishing the signature of a quadratic form.

To see what is meant by this, recall the ingredients necessary for the computation of the signatures of a hermitian from over $Z\pi$. Let π be a finite group and $R\pi$ its real group ring. Any element