PACIFIC JOURNAL OF MATHEMATICS
Vol. 102, No. 1, 1982

CHARACTERIZATIONS OF ARBOROIDS AND DENDRITIC SPACES

T. B. MUENZENBERGER, R. E. SMITHSON, AND L. E. WARD, JR.

In this paper several characterizations of arboroids, arcwise connected dendritic spaces, and a related, wider class of spaces which we call weakly nested are obtained. For example, it is shown that an arcwise connected Hausdorff space is dendritic if and only if it is uniquely arcwise connected and each connected subspace is arcwise connected. These characterizations give considerable insight into the internal structure of such spaces. Also a number of characterizations of topological intervals and trees are given, and an interesting embedding theorem for weakly nested spaces is proved.

1. Introduction. We begin by recalling a few definitions. A *continuum* is a compact, connected Hausdroff space. A space X is *orderable* if it admits a total order $\leqq$ such that all sets of the form $\{x \in X: x < a\}$ and $\{x \in X: a < x\}$ generate the topology, where $a \in X$. It is well-known that an orderable space is completely normal and Hausdorff [1]. An *arc* is a nondegenerate orderable continuum, or, what is the same thing, a continuum with exactly two noncut points, which are called the *endpoints* of the arc. This definition is a departure from and generalization of the classical usage in which an arc is separable. A space is *arcwise connected* if each two distinct points are the endpoints of some arc contained in the space, and it is *uniquely arcwise connected* if that arc is unique. A connected space is *hereditarily unicoherent* if the intersection of any pair of its closed connected subsets is connected.

A connected space is said to be *dendritic* if each pair of distinct points can be separated by some third point. Note that a dendritic space is Hausdorff [2]. A *tree* is a compact dendritic space, and hence a metrizable tree is a *dendrite*.

Knaster has coined the term *dendroid* to mean a metrizable continuum which is hereditarily unicoherent and arcwise connected [3]. Following [14] we use the term *arboroid* for the Hausdorff analog of a dendroid. That is, an arboroid is a continuum which is hereditarily unicoherent and arcwise connected. Although many properties of dendroids carry over directly to arboroids, it is a frequently noted phenomenon that nonmetrizable continua may exhibit pathological qualities not found in the metrizable case. For example, even with the hypothesis of local connectivity, a continuum may fail