PACIFIC JOURNAL OF MATHEMATICS
Vol. 102, No. 2, 1982

ON TWO-STAGE MINIMAX PROBLEMS

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Minimax problems are considered whose admissable sets are given implicitly as the solution sets of another minimax problem. For the solution a parametric method is proposed. Special cases of it are extensions of Courant's exterior penalty method and Tihonov's regularization method of Nonlinear Programming to minimax problems.

In solving quadratic problems explicitly, a representation of modified best approximate solutions of linear equations in Hilbert spaces is given that extends results for the usual case.

1. Introduction. Let X and Y be not empty subsets of real linear topological Hausdorff spaces $\mathscr{X}$ and $\mathscr{Y}$, respectively,

$$f: X \times Y \longrightarrow \boldsymbol{R}, \text{ and } g: X \times Y \longrightarrow \boldsymbol{R}$$

be two real valued functions on $X \times Y$, and denote $X_f \times Y_f$ the solution set of the minimax problem (X, Y, f), i.e.,

$$(x_0, y_0) \in X_f \times Y_f : \Longleftrightarrow \bigwedge_{x \in X} \bigwedge_{y \in Y} f(x, y_0) \leqq f(x_0, y_0) \leqq f(x_0, y) .$$

Note that if (x_1, y_1) and (x_2, y_2) are in $X_f \times Y_f$ then also $(x_1, y_2) \in X_f \times Y_f$, being thus a product set.

Under the assumption that X_f and Y_f are not empty, we give the following

DEFINITION 1. A two-stage minimax problem, in the notation $\mathscr{M}_{g/f}$, is the minimax problem

$$\mathscr{M}_{g/f} := (X_f, Y_f, g/X_f \times Y_f) .$$

Considering $\mathscr{M}_{g/f}$ as a two-person zero-sum game, it describes the following conflict situation: Two antagonists choose independently from each other $x \in X$, resp. $y \in Y$, and the first one gets from the second one the vector-payoff $(f(x, y), g(x, y)) \in \boldsymbol{R}^2$. The preference relation may be induced by the lexicographic order of $\boldsymbol{R}^2$:

(x_1, y_1) is better than (x_2, y_2) for the first (second) player, if $(f(x_1, y_1), g(x_1, y_1))$ is lexicographically greater (smaller) than $(f(x_2, y_2), g(x_2, y_2))$. If the players are cautious, they have to take as optimal strategies the components of a solution of $\mathscr{M}_{g/f}$, provided there exists one.

Many games are of this nature; for example (see §§ 3, 4 and 5