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SYMMETRIC UNBOUNDED LIQUID BRIDGES

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A symmetric unbounded liquid bridge is that surface formed by capillary action when a disc is withdrawn from an infinite reservoir. As do all capillary surfaces, liquid bridges satisfy the condition that mean curvature is proportional to height. The profile curves of the bridge surfaces are parametric extensions of the curves obtained in the symmetric exterior capillary problem. The family of profile curves is shown to have an envelope, and stability and instability criteria are derived relating to this envelope.

Introduction. An unbounded liquid bridge is that surface which is formed when an object is withdrawn a small distance from an infinite pool of liquid, and the liquid is allowed to reach an equilibrium (see Figure 1). The adjective "unbounded" serves to distinguish this surface from the bounded liquid bridges formed by drops of liquid between two parallel plates, or by an object withdrawn from a finite reservoir, neither of which will be studied in this paper.

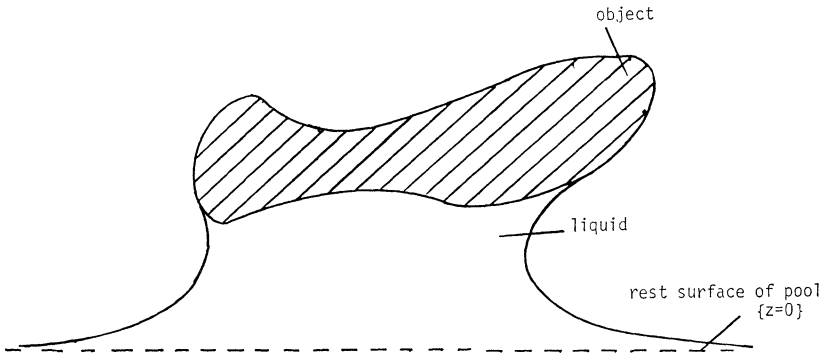

FIGURE 1
The General Unbounded Liquid Bridge

In this paper, I will deal only with the symmetric surface formed in three dimensions when the object withdrawn is a disc parallel to the base plane $\{z = 0\}$. This is a problem in capillarity, so that the shape of the free surface is determined by the interaction of the potential energy due to the lifting of the liquid in a gravitational field, the surface tension of the liquid, and the energy gained by wetting the disc. Specifically, for a given compact set K, define an energy functional acting on sets by:

$$\mathscr{E}_K(A') \equiv \sigma P(A', K) + \rho g \int_{A' \cap K} z - \sigma \beta P(A', \mathscr{D} \cap K) \,. \tag{0.1}$$