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HOMEOMORPHIC CLASSIFICATION OF CERTAIN INVERSE LIMIT SPACES WITH OPEN BONDING MAPS

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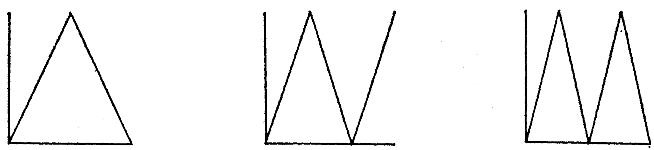
Let $I = [0, 1]$. Let ^{N}f be the Nth degree hat function from I to I. For example, ^{2}f, ^{3}f, and ^{4}f are pictured below:

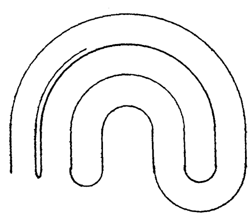


^{2}f ^{3}f ^{4}f

We are interested in classifying the spaces which are inverse limits of the unit interval using these bonding maps. In particular, for a fixed integer $N \geqq 2$, we are interested in classifying (up to homeomorphism) the space D_N, which is $\varprojlim \{I, {}^{N}f\}$. The main result of this paper is:

THEOREM: D_N is homeomorphic to D_M if and only if M and N have the same prime factors.

Overview. Let $D_N = \varprojlim \{I, {}^{N}f\}$. These spaces are often called Knaster continua since D_2 is, in fact, the Knaster Bucket Handle:



$\cong D_2.$

Bellamy [1] and latter Oversteegen-Rogers [2] used D_6 to construct examples of tree-like continua without the fixed point property. It appears improbable that their techniques can be modified to construct a similar example from D_2. This resurrects a question raised in a paper by J. W. Rogers, Jr.—Are there three topologically different D_N's?

The answer, as previously stated is:

THEOREM. *D_N is homeomorphic to D_M if and only if M and N have the same prime factors. (Allowing different bonding maps we will show there are precisely c topologically different Knaster type continua.)*