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AN ARITHMETIC POISSON FORMULA

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Let $a(n)$ denote any arithmetic function. Since most arithmetic functions that are encountered are defined on only the nonnegative integers, we define $a(-n) = a(n)$ for each positive integer n. Our objective is to develop a Poisson type formula for $\sum_{n=-\infty}^{\infty} a(n)f(n)$, where f belongs to a suitable class of functions. We conclude the paper with several applications of our arithmetic Poisson formula.

In another paper [2], the author has used a new technique in contour integration to examine infinite series of the type $\sum_{n=0}^{\infty} a(n)f(n)$, where now f is a suitable rational function. The author's method has been generalized and improved by Krishnaiah and Sita Rama Chandra Rao [5].

Let $b(n)$ be an arithmetic function, and let S denote any subset of the natural numbers. Define, for each positive integer m,

$$a(m, n; S) = \sum_{\substack{d \leqq m \\ d|n \\ d \in S}} b(d) \quad \text{and} \quad a(n; S) = \sum_{\substack{d|n \\ d \in S}} b(d) . \tag{1}$$

Clearly, if $m \geqq n$, $a(m, n; S) = a(n; S)$. If S is the set of all positive integers, we write $a(n; S) = a(n)$. Alternatively, given an arithmetic function $a(n)$, we could define $b(n)$ by

$$b(n) = \sum_{d|n} \mu(d)a(n/d) ,$$

where μ denotes the Möbius function.

There exist several formulations of Poisson's summation formula

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(x)e^{2\pi inx}dx \tag{2}$$

in the literature, where here and in the sequel all doubley infinite sums are interpreted symmetrically, i.e., as $\lim_{N\to\infty} \sum_{n=-N}^{N}$. We shall choose the setting (with slightly stronger hypotheses) from Bellman's book [1, p. 8]. Thus, let $f(x)$ be continuous on $(-\infty, \infty)$, suppose that $f \in L^1(-\infty, \infty)$, and assume that the left and right sides of (2) converge absolutely. Then (2) holds.

THEOREM. *Suppose that f satisfies the conditions of Poisson's formula* (2) *as specified above. Let $a(n)$ and $b(n)$ denote arithmetic functions as related above. Assume also that $\sum_{n=-\infty}^{\infty} a(n)f(n)$ is absolutely convergent. Then*