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THE FINE SPECTRA FOR WEIGHTED MEAN OPERATORS

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In a recent paper [5] the fine spectra of integer powers of the Cesàro matrix were computed. In this paper the fine spectra of weighted mean methods are determined. In most cases investigated, the interior points belong to III_1, the boundary points, except 1, belong to II_2, and 1 and any isolated points belong to III_3, where III_1, II_2, and III_3 are portions of the state space as described in [3].

From Goldberg [3], if $T \in B(X)$, X a Banach space, then there are three possibilities for $R(T)$, the range of T:

(I) $R(T) = X$,
(II) $\overline{R(T)} = X$, but $R(T) \neq X$, and
(III) $\overline{R(T)} \neq X$,

and three possibilities for T^{-1}

(1) T^{-1} exists and is continuous,
(2) T^{-1} exists but is discontinuous,
(3) T^{-1} does not exist.

A weighted mean matrix A is a lower triangular matrix with entries $a_{nk} = p_k/P_n$, where $p_0 > 0$, $p_n \geq 0$ for $n > 0$, and $P_n = \Sigma_{k=0}^n p_k$. The necessary and sufficient condition for the regularity of A is that $\lim P_n = \infty$.

In [2] it was shown that, for any regular weighted mean matrix A, the spectrum, $\sigma(A)$, contains the set $\{\lambda \mid |\lambda - (2-\delta)^{-1}| \leq (1-\delta)/(2-\delta)\} \cup S$, and is contained in the set $\{\lambda \mid |\lambda - (2-\gamma)^{-1}| \leq (1-\gamma)/(2-\gamma)\} \cup \overline{S}$, where $\delta = \limsup p_n/P_n$, $\gamma = \liminf p_n/P_n$, and $S = \{p_n/P_n \mid n \geq 0\}$

We shall first consider those regular weighted mean methods for which $\delta = \gamma$, i.e., for which the main diagonal entries converge.

THEOREM 1. *Let A be a regular weighted mean method such that $\delta = \lim p_n/p_n$ exists. If λ satisfies $|\lambda - (2-\delta)^{-1}| < (1-\delta)/(2-\delta)$ and $\lambda \notin S$, then $\lambda \in \mathrm{III}_1\, \sigma(A)$; i.e., λ is a point of $\sigma(A)$ for which $\overline{R(T)} \neq X$ and T^{-1} exists and is continuous.*

Proof. First of all $\lambda I - A$ is a triangle, hence 1-1. Therefore $\lambda I - A \in 1 \cup 2$.