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EXTREME POINTS IN THE HAHN-BANACH-KANTOROVIČ SETTING

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This paper presents an existence and characterization theorem for the extreme points of the convex set of all extensions of a linear operator from a real vector space into an order complete real vector lattice which are dominated by a sublinear operator. This result is applied to positive extensions, contractions, and dominated invariant extensions.

The paper falls into four sections.

Section 1 is reserved for preliminaries.

In §2 we consider the convex set of all extensions of a linear operator defined on a vector subspace of a real vector space X with values in an order complete real vector lattice Y which are dominated by a sublinear operator P from X into Y. We present a characterization of the extreme points of this set being useful for applications. This part is related to papers of Kutateladze [7], [8] and Portenier [15].

In §3 we give two applications of the preceding result. The first one yields another proof of an existence and characterization theorem due to Lipecki [10], [11] concerning extreme positive extensions of a linear operator which is defined on a subspace of an ordered vector space. The second one yields a new characterization theorem for extreme contractions from a separable Banach space into the space of real valued continuous functions on a compact extremally disconnected space.

In §4 the results of §2 are extended to P-dominated extensions which are positive on a given cone in X, and we apply them to P-dominated extensions which are invariant with respect to a set of mappings from X into X. Furthermore, we obtain a refinement of a dominated extension theorem for positive linear operators due to Luxemburg and Zaanen.

1. Preliminaries. We adhere to the notation of Schaefer's monograph [16]. Throughout X stands for a real vector space, M for its vector subspace and Y for an order complete real vector lattice. P: $X \to Y$ denotes a sublinear mapping, i.e. P is positively homogeneous and subadditive. The space of all linear operators from M into Y is denoted by $L(M, Y)$. Given a vector subspace N of X with $M \subset N$ and $T \in L(M, Y)$, we put

$$E_N(P, T) = \{S \in L(N, Y)\colon S \leq P \mid N \text{ and } S \mid M = T\}.$$