PACIFIC JOURNAL OF MATHEMATICS
Vol 106, No 1, 1983

ALEXANDER DUALITY

WILLIAM JULIAN, RAY MINES AND FRED RICHMAN

The Alexander duality theorem is developed in a manner that is constructive in the sense of Bishop, yielding a constructive Jordan-Brouwer theorem.

0. Introduction. The Alexander duality theorem, which generalizes the Jordan curve theorem and the Jordan-Brouwer theorem, relates the structure of a compact subset of a d-dimensional sphere to that of its complement. The structure of the complement is not completely determined by the structure of the subset. Indeed, the homotopy class of the complement is not determined by the homeomorphism class of the set, as the Alexander horned sphere illustrates. Nevertheless certain connectivity properties of the complement are so determined, such as the number of connected components. This consequence of duality generalizes the Jordan-Brouwer theorem that the complement in the d-sphere of a homeomorph of the $(d-1)$-sphere has two components.

We develop the duality theorem in a manner that is constructive in the sense of Bishop [3], thus generalizing the work in [**1**] on the constructive Jordan curve theorem to obtain a constructive Jordan-Brouwer theorem. The constructive approach requires showing how to use the information given by a homeomorphism of the $(d-1)$-sphere with a subset T of the d-sphere S^d, to construct two points in S^d of positive distance away from T such that any path joining them comes arbitrarily close to T; and, given a point in S^d of positive distance from T, how to construct a path joining it to one of the two points by a polygonal path that is bounded away from T. The strategy, as in the classical situation, is to associate a homeomorphism invariant group $H^{d-1}(X)$ to each compact space X so that if T is a compact subset of S^d and $H^{d-1}(T) = H^{d-1}(S^{d-1})$, then T satisfies the conclusion of the Jordan-Brouwer theorem. To this end we develop a constructive Čech cohomology theory for compact spaces.

In [**6**] a constructive Vietoris homology theory is developed for compact spaces, in which the homology objects are inverse systems of (finitely presented) abelian groups. These more general objects made it possible to get a theory that is both exact and continuous, an impossibility if the objects are required to be abelian groups [**4**; X.4.1]. Moreover some information is lost if we pass to the inverse limit: The first homology *group* of a circle with a missing piece is trivial, but this fact provides no information as to the size of the missing piece. However if we are given