PACIFIC JOURNAL OF MATHEMATICS
Vol. 107, No. 2, 1983

PETTIS INTEGRATION VIA THE STONIAN TRANSFORM

F. DENNIS SENTILLES AND ROBERT F. WHEELER

Let (Ω, Σ, μ) be a finite measure space, and let f be a bounded weakly measurable function from Ω into a Banach space X. Let S be the Stone space of the measure algebra $\Sigma/\mu^{-1}(0)$. Then f induces a continuous map $\hat{f}$: $S \to (X^{**}, \text{weak}^*)$ in a natural way. Criteria for Pettis integrability of f are investigated in the context of this "Stonian transform" $\hat{f}$ of f. In particular, some insight is achieved as to how f can be Pettis integrable without being weakly equivalent to any strongly measurable function. The fine structure of (X^{**}, weak^*) is also examined in this setting.

Let (Ω, Σ, μ) be a complete probability space, X a real Banach space, f: $\Omega \to X$ a function with bounded range. Then f is said to be strongly measurable if there is a sequence (f_n) of measurable X-valued simple functions on Ω such that $\|f_n(w) - f(w)\| \to 0$ a.e. (μ). In this case, the Bochner integral of f over $E \in \Sigma$ is given by $\int_E f\,d\mu = \lim \int_E f_n\,d\mu$, where the integral of a simple function is defined in the obvious way. This integral has been extensively studied; the principal results are recorded in the monograph of Diestel and Uhl [**3**].

There is another concept of measurability and integrability which is less well understood. A bounded function f: $\Omega \to X$ is *weakly measurable* if $x^* \circ f$ is a measurable scalar-valued function on Ω for each $x^* \in X^*$, the dual space of X. If $E \in \Sigma$, then the *Dunford integral* of f over E is the member x_E^{**} of X^{**} defined by $x_E^{**}(x^*) = \int_E x^* \circ f\,d\mu$. If x_E^{**} is a member of X, for each $E \in \Sigma$, then f is said to be *Pettis integrable*, and we write $x_E = (P)\int_E f\,d\mu$ for the integral. This notion was originally studied by Pettis [**18**]; again the classical results are recorded in [**3**]. Recently there has been considerable progress in the study of the Pettis integral [**6**, **7**, **8**, **9**, **10**, **24**, **27**, **28**].

In this work we analyze the nature of Pettis integrability by converting measurability into continuity. Let S be the Stone representation space of the measure algebra $\Sigma/\mu^{-1}(0)$, and let $h \to \hat{h}$ be the usual isometry between $L^\infty(\Omega, \Sigma, \mu)$ and $C(S)$. Then the *Stonian transform* of f: $\Omega \to X$ is the function $\hat{f}$: $S \to X^{**}$ defined by $\langle \hat{f}(s), x^* \rangle = \widehat{x^* \circ f}(s)$, for all $x^* \in X^*$ [**26**]. Clearly $\hat{f}$ is continuous when X^{**} is given the weak*-topology. We consider the questions: (a) Can $\hat{f}$ actually be computed in a meaningful