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UNIFORM DISTRIBUTION IN SUBGROUPS OF THE BRAUER GROUP OF AN ALGEBRAIC NUMBER FIELD

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We construct subgroups of the Brauer group of an algebraic number field whose member classes have Hasse invariants satisfying a rigid arithmetic structure — that of (relative) uniform distribution. After obtaining existence and structure theorems for these subgroups, we focus on the problem of describing algebraic properties satisfied by the central simple algebras in these subgroups. Key results are that splitting fields are determined up to isomorphism, and there exists a distinguished subgroup of central automorphisms which can be extended.

1. Introduction. Let K be an algebraic number field, and let $[A]$ denote the class of the finite dimensional central simple K-algebra A in the Brauer group $B(K)$ of K. The class $[A]$ is determined arithmetically by its Hasse invariants at the primes of K. Algebraic properties of A often impose severe but interesting arithmetic properties on its invariants. As evidence we cite the important work of M. Benard and M. Schacher [2] concerning the invariants when $[A]$ is in $S(K)$ the Schur subgroup of K, and the surprising result of G. Janusz [4] obtained in considering the problem of when an automorphism of K extends to A.

In this paper we offer a construction which gives rise to subgroups of $B(K)$ whose member classes have invariants which possess a rigid arithmetic structure — that of uniform distribution — then search for corresponding algebraic properties. Our construction is modeled after one given by R. Mollin [5] to study subgroups of $B(K)$ which contain $S(K)$.

In §2 we present our construction and establish a series of results which culminate in an existence theorem. In §3, we consider questions concerning the structure of our subgroups. In §4, we present our main result, which shows that the classes we consider have splitting fields which are determined up to isomorphism, and in fact we characterize such classes. Our final section incorporates the aforementioned work of Janusz to give a wholly different algebraic description to our classes.

For the general theory of central simple algebras we refer the reader to [1]. If $[A] \in B(K)$, then A is a matrix ring over a unique division ring D and the index of $[A]$ written ind$[A]$ is $\sqrt{[D:K]}$. Moreover, if ind$[A]$