PACIFIC JOURNAL OF MATHEMATICS
Vol 108, No. 1, 1983

L^p-BOUNDEDNESS OF THE MULTIPLE HILBERT TRANSFORM ALONG A SURFACE

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For an appropriate surface σ in R^n, we prove that the multiple Hilbert transform along σ is a bounded operator on $L^p(R^n)$, for p sufficiently close to 2. Our analysis of this singular integral operator proceeds via Fourier transform techniques—that is, on the "multiplier side"—with applications of Stein's analytic interpolation theorem and the Marcinkiewicz multiplier theorem. At the heart of our argument we have estimates of certain trigonometric integrals.

CONTENTS

I. Introduction. The present work continues that of Fabes, Nagel, Rivière, Stein, and Wainger on singular integral operators associated with curves or surfaces in R^n. For an appropriate curve γ: $R \to R^n$ we define the Hilbert transform H along γ by the principal value integral $Hf(x) = \text{p.v.}\int_{-\infty}^{\infty} f(x - \gamma(t))\,dt/t$, $x \in R^n$, $f \in C_c^\infty(R^n)$. In the papers **[1]** of Fabes; **[11]** and **[12]** of Stein and Wainger; **[2]** and **[3]** of Nagel, Rivière, and Wainger; and **[5]** of Nagel and Wainger, it has been shown that for a variety of curves γ, the operator H is bounded on $L^2(R^n)$, or on $L^p(R^n)$ for some or all p in the range $1 < p < \infty$; on the other hand, there are C^∞ curves γ for which H fails to be bounded even on $L^2(R^n)$.

Nagel and Wainger **[6]** have introduced the multiple Hilbert transform along σ, defined for $f \in C_c^\infty(R^n)$ and $x \in R^n$ by

$$(1)\quad Tf(x) = \lim_{\substack{\varepsilon\to 0\\ N\to\infty}} T_{\varepsilon,N}f(x) = \lim_{\substack{\varepsilon\to 0\\ N\to\infty}} \int \cdots \int_{\substack{\varepsilon<|t_i|<N\\ (1\leq i\leq k)}} f(x-\sigma(t))\frac{dt_1}{t_1}\cdots\frac{dt_k}{t_k}.$$

Here, σ is the k-surface in R^n given for $t = (t_1,\ldots,t_k) \in R^k$ by $\sigma(t) = (t_1,\ldots,t_k,\gamma_1(t),\ldots,\gamma_l(t))$ where $n = k + l$ and $\gamma_j(t) = \prod_{i=1}^k |t_i|^{\alpha_{i,j}}$. Nagel and Wainger showed that T is bounded on $L^2(R^n)$ if the exponents $\alpha_{i,j}$ are appropriately restricted. Our proof that T is bounded on $L^p(R^n)$ for p