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ANALYTIC LINEARIZATION OF THE KORTEWEG-DE VRIES EQUATION

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We prove that the *KdV* equation is linearized by an analytic function, which is projectively analytically invertible. The Cauchy problem for the *KdV* equation is entirely solved by this fact. The non-linear superposition principle is a trivial consequence of convexity for the image of the linearization operator.

1. Introduction. Since the discovery [7] of the inverse scattering formalism for the *KdV* equation

$$\frac{\partial}{\partial t}u(t,x)+\frac{\partial^3}{\partial x^3}u(t,x)-6u(t,x)\frac{\partial}{\partial x}u(tx)=0, \tag{1.1}$$

$$t,x,u(t,x)\in\mathbf{R},$$

it is known, given a certain class of solution for the linear equation

$$\frac{\partial}{\partial t}v(t,x)+\frac{\partial^3}{\partial x^3}v(t,x)=0$$

how to construct solutions of equation (1.1). However, it is not clear how this reduces the Cauchy problem for the *KdV* equation, on a given space of initial conditions, into that of the above linear equation. The Cauchy problem for (1.1) has been solved by direct functional analysis methods on the Sobolev space H^3, ([**11**]), and on H^2, ([**1**]). The inverse scattering formalism has been used to solve the Cauchy problem on $S(\mathbf{R})$, ([**12**]) and for sufficiently rapidly decreasing C^3 initial conditions ([**3**]), where in both cases, two linear problems are associated with the *KdV* equation.

To formulate the problem of linearization of the non linear Cauchy problem, it is convenient to give topological vectorspaces of initial conditions and of solutions for the non linear resp. for the linear problem. In this context it is possible to give a precise meaning to the concept of linearization (see [**5**]). What we want to show in this paper is that the linearization program defined in [**6**], entirely goes through and solves the initial value problem for the *KdV* equation. We stress the fact that this approach is straightforward (in contrast to the inverse scattering formalism), when the spaces of initial conditions are given. The inverse