PACIFIC JOURNAL OF MATHEMATICS
Vol. 108, No. 1, 1983

ENDOMORPHISMS OF RANK ONE MIXED MODULES OVER DISCRETE VALUATION RINGS

WARREN MAY AND ELIAS TOUBASSI

Mixed modules over a discrete valuation ring with torsion-free rank one and simply presented torsion submodules are considered. It is proved that every isomorphism of endomorphism algebras of such modules is induced by an isomorphism of the modules.

We begin by recalling some relevant results. A simple antecedent of the Noether-Skolem theorem states that if V is a vector space over a field F, then every automorphism of the endomorphism algebra of V is inner. R. Baer proved a similar result for bounded p-primary groups which was subsequently generalized by Kaplansky to torsion modules over discrete valuation rings. Fix a discrete valuation ring R (and choose a prime element p). If M is an R-module, we denote the endomorphism algebra of M by $E(M)$. In this paper, we shall prove the following

THEOREM. *Let M be an R-module of torsion-free rank one with simply presented torsion submodule. If N is an R-module of torsion-free rank one, then every algebra isomorphism Φ: $E(M) \to E(N)$ is induced by an isomorphism θ: $M \to N$ such that $\Phi(\alpha) = \theta\alpha\theta^{-1}$ for every $\alpha \in E(M)$.*

COROLLARY. *Every algebra automorphism of $E(M)$ is inner.*

We remark that the isomorphism θ is not obtained by applying the familiar invariants for rank one modules with simply presented torsion, but is constructed directly. We have restricted our attention to modules over discrete valuation rings in view of an example in [5] which suggests that the theorem does not generalize to modules over principal ideal domains which have infinitely many primes.

To fix notation, let K denote the quotient field of R. Moreover, we shall always let M and N denote R-modules of torsion-free rank one, with respective torsion submodules $T(M)$ and $T(N)$. If Φ: $E(M) \to E(N)$ is an R-algebra isomorphism, then there exists an isomorphism

$$\text{(A)} \qquad \phi\colon T(M) \to T(N)$$