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DIVERGENCE OF COMPLEX RATIONAL APPROXIMATIONS

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General rational interpolations, orthogonal-Padé approximations and best rational real approximations are shown to diverge as badly as classical Padé approximants. The examples also show known convergence results to be best possible in a strong sense.

1. Introduction. In [3], the author used extensions of Wallin's methods [**10**] to show that the well known Nuttall-Pommerenke theorem on convergence in capacity of Padé sequences is substantially best possible. One might expect that general rational interpolations with free poles, should fare better than classic Padé approximants, at least inside the closure of the interpolation points. Surprisingly they do not.

In this note, a new method is used to establish counterexamples to extension of known convergence results for (i) rational interpolants (ii) Padé-orthogonal approximations (iii) best rational real approximations. More specifically, it is shown that diagonal and non-diagonal rational sequences formed from entire functions may diverge in the limit on given σ-compact sets of capacity zero, and that diagonal sequences formed from functions with finite radius of analyticity may diverge in the limit on sets whose intersection with every open ball has positive area. Even in the classic Padé case, the latter example is more complete than Theorem 3 in [3]. It also settles conclusively a problem posed by Goncar[1].

2. Notation. (i) Throughout L, L_i, M, M_i, N, N_i denote positive integers and

$$T(i) = L_i + M_i + 1. \tag{2.1}$$

Further I is a bounded real interval and for any function $f\colon I \to \mathbf{C}$, let $\|f\| = \sup\{|f(t)| : t \in I\}$. Also let $\|I\| = \sup\{|t| : t \in I\}$.

(ii) Given any integer $n \geq 1$, $\mathcal{P}_n$ is the class of polynomials of degree n with 1 as (leading) coefficient of z^n. Also $\mathcal{P}_0 = \{1\}$.

[1] A. A. Goncar, *On the convergence of generalized Padé approximants to meromorphic functions*, Math. USSR Sbornik, **27** (1975), 503–514. On page 504: "If $D(f)$ is a disc of finite radius..."