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AN ORDERING FOR THE BANACH SPACES

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A binary relation will be defined on the class of all Banach spaces. The relation is transitive and symmetric, so it is natural to call it an "ordering". (The definition also makes sense for locally convex spaces with good duality properties, but this will not be pursued here.) Many of the elementary properties of the ordering are spelled out. Although some connections with Pettis integration and unique preduals have been found, the usefulness of this ordering in Banach space theory remains to be determined.

Notation and terminology used in this paper generally matches Dunford and Schwartz [**4**], Chapters IV and VI. More recent results in Banach space theory will usually be quoted from Lindenstraus and Tzafriri [**11**] or from Diestel and Uhl [**3**]. If $\mathfrak{X}$ is a Banach space, its dual will be denoted $\mathfrak{X}^*$, its bidual $\mathfrak{X}^{**}$. The subset of $\mathfrak{X}^{**}$ canonically identified with $\mathfrak{X}$ will simply be written $\mathfrak{X}$.

DEFINITION. Let $\mathfrak{X}$ and $\mathfrak{Y}$ be Banach spaces. Then $\mathfrak{X} \prec \mathfrak{Y}$ means

$$\mathfrak{X} = \bigcap T^{**-1}[\mathfrak{Y}],$$

where the intersection is over all bounded linear operators $T\colon \mathfrak{X} \to \mathfrak{Y}$.

The definition can be rephrased as follows: $\mathfrak{X} \prec \mathfrak{Y}$ if and only if any $\alpha \in \mathfrak{X}^{**}$, such that $T^{**}(\alpha) \in \mathfrak{Y}$ for all bounded linear operators $T\colon \mathfrak{X} \to \mathfrak{Y}$, must be in $\mathfrak{X}$.

A single operator T with $\mathfrak{X} = T^{**-1}[\mathfrak{Y}]$ has been called a *Tauberian operator* (see [**10**]). If there exists a Tauberian operator $\mathfrak{X} \to \mathfrak{Y}$, then $\mathfrak{X} \prec \mathfrak{Y}$, but not conversely.

Following [**9**], where the case $\mathfrak{Y} = l_1$ is considered, we define the $\mathfrak{Y}$*-frame* (cadre) of $\mathfrak{X}$ by

$$\mathfrak{F}(\mathfrak{X}, \mathfrak{Y}) = \bigcap T^{**-1}[\mathfrak{Y}],$$

with intersection over all operators $T\colon \mathfrak{X} \to \mathfrak{Y}$. Then $\mathfrak{F}(\mathfrak{X}, \mathfrak{Y})$ is a Banach space, $\mathfrak{X} \subseteq \mathfrak{F}(\mathfrak{X}, \mathfrak{Y}) \subseteq \mathfrak{X}^{**}$. One extreme possibility is $\mathfrak{F}(\mathfrak{X}, \mathfrak{Y}) = \mathfrak{X}^{**}$,