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# FREE INTERPOLATION FOR HOLOMORPHIC FUNCTIONS REGULAR TO THE BOUNDARY

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**We consider the class Lip $s$ of holomorphic functions in the unit disc satisfying a Lipschitz condition of order $s$, $0 < s \leq 1$ and the class $A^1$ of holomorphic functions $f$ such that the derivatives $f'$ belong to the disc algebra. In this paper we give the complete characterization of the *interpolation sets* for these classes, thus completing the previous works of J. Bruna, E. M. Dyn'kin and E. P. Kronstadt.**

**I. Introduction and statement of results.** This paper deals with interpolation problems in classes of holomorphic functions in the unit disc $D$ of the complex plane that have some regularity up to the boundary $T$. We denote by $A$ the disc algebra. We will consider the following classes included in $A$

$$\text{Lip}\, s = \{f \in A; |f(z) - f(w)| \leq \text{const}\, |z - w|^s, z, w \in D\}, \qquad 0 < s \leq 1,$$

$$A^1 = \{f \in A;\, f' \in A\}.$$

We could consider as well more regular classes Lip $s$ for $s > 1$ or $A^p$ with $p = 2, 3, \ldots$. Our results and methods also apply to them with minor changes. To simplify the development we confine ourselves to the cases indicated.

For a given closed set $E \subset \overline{D}$, we denote by $\text{Lip}_s(E)$ the space of functions on $E$ satisfying a Lipschitz condition of order $s$. For $C^1(E)$ we choose the Whitney definition. That is, $C^1(E)$ consists of the 1-jets $\phi = (\phi, \phi_z, \phi_{\bar z})$, where $\phi$, $\phi_z$, $\phi_{\bar z}$ are continuous functions on $E$ such that

$$|\phi(w) - \phi(z) - \phi_z(z)(w - z) - \phi_{\bar z}(z)(\bar w - \bar z)| = o(|w - z|)$$

uniformly in $w, z \in E$. By the Whitney extension theorem ([7], pg. 5) one can think of $\phi$ being the jet induced by a function $\phi$ in $C^1(\mathbf{C})$ with compact support (and then $\phi_z = \partial\phi/\partial z$, $\phi_{\bar z} = \partial\phi/\partial \bar z$ on $E$).

DEFINITIONS. $E$ is called an *interpolation set for* Lip $s$ if given any $\phi \in \text{Lip}_s(E)$ there exists $f \in \text{Lip}\, s$ such that $f = \phi$ on $E$. Similarly, we say that $E$ is an *interpolation set for* $A^1$ if given $\phi \in C^1(E)$ with $\phi_{\bar z} = 0$