PACIFIC JOURNAL OF MATHEMATICS
Vol. 109, No. 2, 1983

A REMARK ON THE ISOTOPY CLASSES OF DIFFEOMORPHISMS OF LENS SPACES

W. C. HSIANG AND B. JAHREN

Let L be a lens space of dimension at least 5 and with fundamental group of odd order. In this paper we reduce the computation of $\pi_0(\mathrm{Diff}\, L)$ to algebraic K-theory and homotopy theory. The approach is via parametrized surgery theory, as developed by the first author and R. Sharpe.

I. Introduction and statement of the result. In this note, we shall follow an idea of Browder's [**1**] and use the braid of groups of [**8**] (cf. [**2**]) to describe $\pi_0(\mathrm{Diff}\, L^{2n+1})$ $(n \geq 2)$ where L^{2n+1} is a lens space with $\pi = \pi_1(L^{2n+1}) = \mathbf{Z}_l$ a cyclic group of odd order.

Let $\mathcal{P}(L^{2n+1}) = \mathrm{Diff}(L^{2n+1} \times [0,1];\ L^{2n+1} \times 0)$ be the pseudo-isotopy space of diffeomorphism of L^{2n+1}. Turning upside down, we have an involution '$-$' on $\pi_0\mathcal{P}(L^{2n+1})$ [**6**]. The quotient group

$$\pi_0\mathcal{P}(L^{2n+1}) / \{x - \bar{x}\}$$

can be described as a cobordism group $\pi_0(\mathcal{B}(L^{2n+1}))$ as follows. An object $\alpha = (W(\alpha), f)$ is a diagram

$$\begin{tikzcd}
 & L^{2n+1} \arrow[dl, "j"'] \arrow[dr, "k"] & \\
(W, \partial W, M) \arrow[rr, "f"] \arrow[dr, "p"'] & & L^{2n+1} \times (D^2, \partial D^2, 1) \arrow[dl, "p_0"] \\
 & (D^2, \partial D^2, 1) &
\end{tikzcd} \tag{1}$$

satisfying the following conditions:

(i) 1 is the base point of ∂D^2 and the upper triangle is commutative with j a diffeomorphism and k the standard identification of L^{2n+1} with $L^{2n+1} \times 1$,

(ii) f is a simple homotopy equivalence,

(iii) the lower triangle is commutative with p_0 the projection onto the second factor, and $L^{2n+1} \xrightarrow{j} \partial W \xrightarrow{p} \partial D^2$ is a smooth fibration.