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LOCALLY CONVEX SPACES OF NON-ARCHIMEDEAN VALUED CONTINUOUS FUNCTIONS

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We study the space $C(X, K, \mathcal{P})$ of all continuous functions from the ultraregular space X into the non-Archimedean valued field K with topology of uniform convergence on a family $\mathcal{P}$ of subsets of the Z-repletion of X. We characterize the bornological space associated to $C(X, K, \mathcal{P})$, semi-bornological spaces $C(X, K, \mathcal{P})$, reflexivity and semi-reflexivity both for spherically complete and non-spherically complete K.

1. Introduction. Throughout this paper, K is a complete non-trivially non-Archimedean valued field and X is an ultraregular (= zero-dimensional Hausdorff) space. Then $X \subseteq v_K X \subseteq v_0 X \subseteq \beta_0 X$ where $v_K X$, $v_0 X$ and $\beta_0 X$ are the K-repletion, **Z**-repletion and Banaschewski compactification of X, respectively. If K has nonmeasurable cardinal, then $v_K X = v_0 X$ [**1**, Theorem 15].

The set $|K| = \{|\lambda| : \lambda \in K\}$ is provided with a topology in which all points are discrete, except for 0, whose neighborhoods are the usual ones. $|K|$ is a complete metric space under the metric

$$d(x, y) = \begin{cases} \max(x, y) & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases}$$

Hence $|K|$ is **Z**-replete [**1**, Theorem 9], so $|f|$ can be extended continuously over the whole of $v_0 X$ whenever f belongs to the vector space $C(X, K)$ of all continuous functions from X into K.

A set $A \subseteq v_0 X$ is called bounding if $\|f\|_A := \sup_{x \in A} |f|(x) < \infty$ for all $f \in C(X, K)$. We omit the relatively easy proof of the following:

PROPOSITION 1. *The following are equivalent for $A \subseteq v_0 X$:*

(i) *A is bounding.*

(ii) *Every $g \in C(v_0 X, |K|)$ is bounded on A.*

(iii) *If $(U_i)_{i=1}^{\infty}$ is a partition of $v_0 X$ in open-and-closed subsets, then $U_i \cap A = \varnothing$ for all but finitely many i.*

(iv) *If $g \in C(v_0 X, |K|)$, then $g(A)$ is compact in $|K|$.*

(v) *If $g \in C(v_0 X, |K|)$, then $g(A)$ is relatively compact in $|K|$.*

(vi) *$\overline{A}^{v_0 X}$ is compact.*