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CIRCLE ACTIONS ON HOMOTOPY SPHERES WITH CODIMENSION 4 FIXED POINT SET

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In this paper we give a complete equivariant classification of smooth S^1 actions on homotopy spheres with codimension 4 fixed point set and point out a relationship with a natural generalization of the twist-spinning process for knots.

Semifree S^1 actions on homotopy spheres with codimension 4 fixed point set have been classified by J. Levine; so we concentrate on actions with exceptional orbits. There are some obvious linear models for these actions. Let ξ be the standard generator of the complex representation ring of S^1. Then in some sense the actions with one exceptional orbit type are modeled after $\xi^k \oplus \xi \oplus 0$ and those with two exceptional orbit types are modeled after $\xi^k \oplus \xi^m \oplus 0$. Let $\mathbb{S}_k^n$ denote the set of diffeomorphism classes of pairs (Σ_k^{n-1}, Δ) where Σ^{n-1} is a homotopy $(n-1)$-sphere and Δ_k is a smooth Z_k-acyclic orientable codimension 2 submanifold with boundary an integral homology sphere. Similarly, for relatively prime integers k and m, let $\mathbb{S}_{k,m}^n$ denote the set of triads $(\Sigma^{n-1}; \Delta_k, \Delta_m)$ where Δ_k and Δ_m are respectively Z_k and Z_m-acyclic orientable codimension 2 smooth submanifolds meeting tangentially such that $\partial\Delta_k = \partial\Delta_m = \Delta_k \cap \Delta_m$ is an integral homology sphere. In these two cases the classification theorem states that actions on homotopy n-spheres with one exceptional orbit type Z_k, or two exceptional orbit types Z_k and Z_m are in 1-1 correspondence with $\mathbb{S}_k^n$ and $\mathbb{S}_{k,m}^n$. These 1-1 correspondences are realized by associating with an S^1 action on a homotopy n-sphere its structured orbit space and viewing Δ_k and Δ_m as the images in the orbit space of the fixed point sets of Z_k and Z_m.

That these two types of actions do not comprise all S^1 actions on (homotopy) spheres with codimension 4 fixed point set was shown by E. V. Stein in answer to a question of Frank Raymond. It turns out that all these other actions correspond in a 1-1 fashion via their structured orbit spaces to the set $\mathcal{T}_{k,m}^n$ of diffeomorphism classes of triads $(\Sigma^{n-1}, \Delta_k, \Delta_m)$ as in the definition of $\mathbb{S}_{k,m}^n$, except that the interiors of Δ_k and Δ_m intersect transversely in a (perhaps disconnected) n–5 manifold without boundary. This intersection manifold corresponds to the image in the orbit space of the exceptional orbits of type Z_{km}. The intersecting aspect