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RATIONAL SURGERY CALCULUS: EXTENSION OF KIRBY'S THEOREM

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Every 3-dimensional closed orientable manifold may be constructed by surgery on the 3-sphere as introduced by M. Dehn. We are concerned in this paper with answering the question: when do two surgery schemes actually produce the same 3-manifold?

Kirby [4] described certain geometric moves — a calculus of links — and showed that two "framed links" in S^3 produce homeomorphic surgery manifolds if and only if they are related by these moves. But not all surgeries are permitted in the Kirby calculus. Only those which correspond to attaching 2-handles to the 4-ball are considered, since Kirby's techniques are 4-dimensional.

EXAMPLE. Dehn considered the homology spheres determined by the surgery data

where n is an integer (notation of [6], also explained below.) Except for $n = \pm 1$, these constructions are not considered in the Kirby calculus. In continuations of this example we will show that these spaces are all distinct. One can, of course, construct these manifolds using surgeries permitted in the Kirby calculus, but one is obliged to use different knots K_n for each n, which we will construct.

In this paper we prove a generalization of Kirby's theorem which applies to the entire class of Dehn surgeries on S^3. Instead of Kirby's calculus, we use the "rational" link calculus introduced (independently) in [6]. Our main result states that different surgery data (= links with rational coefficients) yield the same oriented manifold if and only if the