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THE SPECIES OF BORDERED KLEIN SURFACES WITH MAXIMAL SYMMETRY OF LOW GENUS

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A compact bordered Klein surface of genus $g \geq 2$ is said to have *maximal symmetry* if its automorphism group is of order $12(g-1)$, the largest possible. For each value of the positive integer g there are, of course, several different topological types of bordered surfaces of genus g; each distinct topological type is called a *species* of the genus g. Here we classify the species of bordered Klein surfaces with maximal symmetry of genus $g \leq 40$; there are 32 species in 18 different genera. We also classify the species with maximal symmetry that have no more than 5 boundary components. To aid in the classification two group-theoretic constructions that give new surfaces with maximal symmetry and a family of M^*-groups are introduced. We also establish several general results about the species of a surface with maximal symmetry. In particular we show that if X is a non-orientable bordered surface with maximal symmetry and solvable automorphism group, then the genus of X is odd.

0. Introduction. Let X be a compact Klein surface [**1**] of (algebraic) genus $g \geq 2$. Then the group of automorphisms of X is finite, and it is well-known just how large this group can be. The size of the best possible upper bound depends, however, on the topological type of the surface X.

If X is orientable and without boundary, then X is a classical Riemann surface and has at most $168(g-1)$ automorphisms (including the orientation-reversing ones). This of course is just twice the bound Hurwitz obtained in his fundamental paper [**7**]. Recent research has studied the values of g for which these bounds are attained and the structure of the associated automorphism groups. Most work has concentrated on the groups of orientation-preserving automorphisms (for example, see [**2**] and [**13**]), but the full groups were considered in [**16**]. These groups cannot be solvable [**13**, p. 19]. Hurwitz's bound $84(g-1)$ is attained for infinitely many values of g, but these values have not been classified. The first four values are 3, 7, 14, and 17, and these are the only ones that are not greater than 40 [**13**, p. 38].

If the surface X is non-orientable but still without boundary, then the order of the automorphism group is at most $84(g-1)$. This case has received a good deal less attention but has been studied in [**15**] and [**5**].