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COMPACT ELEMENTS OF WEIGHTED GROUP ALGEBRAS

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For a locally compact group G let $L^1(G, \omega\lambda)$ be the weighted group algebra. We characterize elements $g \in L^1(G, \omega\lambda)$ for which the operator $T_g(f) = f * g$ $(f \in L^1(G, \omega\lambda))$ is compact. We conclude a result due to S. Sakai that if G is a locally compact non-compact group, then 0 is the only compact element of $L^1(G, \lambda)$, and a result due to C. Akemann that if G is a compact group, then every element of $L^1(G, \lambda)$ is compact.

In a recent paper ([**2**], Theorem 2.2) W. G. Bade and H. G. Dales, among other things, characterize compact elements of $L^1(R^+, \omega)$. Niels Grønbæk, in his Ph.D. thesis ([**6**] Proposition 2.4), for a large class of semigroups (including cancellation semigroups) characterizes compact elements of the discrete weighted semigroup algebras.

In this paper, we characterize the compact elements of the weighted group algebras of locally compact groups. S. Sakai has proved that if G is a locally compact non-compact group, then 0 is the only compact element of $L^1(G, \lambda)$, (see [**10**], Theorem 1), and C. Akemann has proved that if G is a compact group, then every element of $L^1(G, \lambda)$ is compact ([**1**], Theorem 4). These two results will immediately follow from our characterization of the compact elements of the weighted group algebras. Also, a technique somewhat similar to ours provides other proofs for the Bade-Dales theorem (for bounded ω) and the theorem of Grønbæk.

By a weight function on a locally compact group G we mean a positive and continuous function ω on G such that $\omega(st) \leq \omega(s)\omega(t)$ $(s, t \in G)$. If λ is a left Haar measure on G and ω is a weight function on G, we set

$$L^1(G, \omega\lambda) = \left\{f\colon \|f\| = \int_G |f(t)|\omega(t)\, d\lambda(t) < \infty\right\}.$$

Then, $L^1(G, \omega\lambda)$ is a Banach space: as usual, we equate functions equal λ almost everywhere. Under the convolution product defined by the equation

$$(f * g)(x) = \int_G f(xy^{-1})g(y)\, d\lambda(y) \qquad (f, g \in L^1(G, \omega\lambda))$$