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3-MANIFOLDS WITH SUBGROUPS $Z \oplus Z \oplus Z$ IN THEIR FUNDAMENTAL GROUPS

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In this paper we characterize those 3-manifolds M^3 satisfying $\mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z} \subseteq \pi_1(M)$. All such manifolds M arise in one of the following ways: (I) $M = M_0 \# R$, (II) $M = M_0 \# R^*$, (III) $M = M_0 \cup_\partial R^*$. Here M_0 is any 3-manifold in (I), (II) and any 3-manifold having P^2 components in its boundary in (III). R is a flat space form and R^* is obtained from R and some involution $\iota\colon R \to R$ with fixed points, but only finitely many, as follows: if $C_1, \ldots, C_n$ are disjoint 3-cells around the fixed points then R^* is the 3-manifold obtained from $(R - \operatorname{int}(C_1 \cup \cdots \cup C_n))/\iota$ by identifying some pairs of projective planes in the boundary.

1. Introduction. In [**1**] it was shown that the only possible finitely generated abelian subgroups of the fundamental groups of 3-manifolds are Z_n, $Z \oplus Z_2$, Z, $Z \oplus Z$ and $Z \oplus Z \oplus Z$. The purpose of this paper is to characterize all M^3 satisfying $Z \oplus Z \oplus Z \subseteq \pi_1(M)$.

To explain this characterization recall that the Bieberbach theorem (see Chapter 3 of [**8**]) implies that if M is a closed 3-dimensional flat space form then $Z \oplus Z \oplus Z \subseteq \pi_1(M)$. We let $M_1, \ldots, M_6$ denote the 6 compact connected orientable flat space forms in the order given on p. 117 of [**8**]. Similarly $N_1, \ldots, N_4$ will denote the non-orientable ones. For explicit descriptions see §2. One of the main theorems from [**3**] is

(1.1) THEOREM. *The only space forms from the orientable ones $M_1, \ldots, M_6$ which admit involutions having fixed points, but only finitely many, are M_1, M_2, M_6. Moreover these involutions are unique up to conjugacy and have* 8, 4, 2 *fixed points respectively.*

If $\iota\colon M_i \to M_i$, $i = 1$, 2 or 6, is such an involution and $x_1, \ldots, x_n$ are the fixed points ($n = 8, 4, 2$) then there are disjoint 3-cells $C_1, \ldots, C_n$ so that

$$x_i \in \operatorname{int} C_i \quad \text{and} \quad \iota(C_i) = C_i, \qquad 1 \le i \le n.$$

We let M_i^* denote the orbit manifold $M_i - \operatorname{int}(C_1 \cup \cdots \cup C_n)/\iota$. Thus ∂M_1^* consists of 8 projective planes, ∂M_2^* consists of 4 and ∂M_6^* has 2. Canonical presentations of M_i^*, $i = 1, 2, 6$, are given in [**3**]. By making identifications of pairs of such projective planes in ∂M_i^* we obtain new manifolds still containing $Z \oplus Z \oplus Z$ in their fundamental groups. We