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COMPACT QUOTIENTS BY C*-ACTIONS

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Let X be a compact normal complex space on which $\mathbf{C}^*$ acts 'in a nice manner'. We describe all invariant open subsets U of X such that the holomorphic map $U \to U/\mathbf{C}^*$ of U onto the categorical quotient for the category of compact complex spaces, $U/\mathbf{C}^*$, is locally Stein. The description depends on a partial ordering of the fixed point components which arises from the Bialynicki-Birula decompositions of X.

Introduction. Let $\rho\colon T \times X \to X$ be a meromorphic action, (cf. §1), of $T = \mathbf{C}^*$ on an irreducible compact normal complex analytic space X. Such an action is said to be locally linearizable if and only if given any $x \in X$ there is a T-invariant neighborhood V of x and a proper T-equivariant holomorphic embedding of V into $\mathbf{C}^n$ with T acting linearly on $\mathbf{C}^n$.

In this paper we solve the following problem:

Describe all T-invariant Zariski open subsets U of X, such that U/T is a compact complex analytic space and $U \to U/T$ is a semi-geometric quotient (i.e. a categorical quotient which is locally Stein cf. (1.8)).

This problem has been solved by A. Bialynicki-Birula and A. Sommese, [**B** $-$ **B** $+$ **S**], under the above setting when U contains no fixed points and by A. Bialynicki-Birula and J. Swiecieka, [**B** $-$ **B** $+$ **Sw**], when the action is algebraic and X is a compact algebraic variety.

As in [**B** $-$ **B** $+$ **S**], our description of semi-geometric quotients $U \to U/T$ is intimately linked to a certain partial ordering of the fixed point components $F_1,\ldots,F_r$. So that we can state our results precisely we shall introduce the following notation. We assume that all analytic spaces are Hausdorff, reduced and have countable topology.

Let $\{F_1,\ldots,F_r\}$ be the connected components of the fixed point set of T, X^T. Define ϕ^+, $\phi^-\colon X \to X^T$ by $\phi^+(x) = \lim_{t\to 0} tx$ and $\phi^-(x) = \lim_{t\to\infty} tx$, respectively.

Let $X_i^+ = \{x \in X \mid \phi^+(x) \in F_i\}$, $i = 1,\ldots,r$, and $X_i^- = \{x \in X \mid \phi^-(x) \in F_i\}$, $i = 1,\ldots,r$.

An index i is said to be *directly less than* an index j if $C_{ij} = (X_i^+ - F_i) \cap (X_j^- - F_j) \neq \varnothing$. We say that i is *less than* j, denoted $i < j$, if there exists a sequence $i = i_0,\ldots,i_k = j$ such that i_l is directly less than i_{l+1} for $l = 0,\ldots,k-1$. This relation forms an ordering of the indices $\{1,\ldots,r\}$.