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KRULL DIMENSION OF SKEW-LAURENT EXTENSIONS

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A precise formula is derived for the (noncommutative) Krull dimension of a skew-Laurent extension $R[\theta_1^{\pm 1},\dots,\theta_u^{\pm 1}]$, where R is a commutative noetherian ring of finite Krull dimension, equipped with u commuting automorphisms $\sigma_1,\dots,\sigma_u$. The formula is given in terms of heights and automorphian dimensions of prime ideals of R, where the automorphian dimension of a prime ideal P is a positive integer that measures the invariance of P relative to products of powers of the σ_i. As part of the development of this formula, the Krull dimension of a skew-Laurent extension $R[\theta_1^{\pm 1}]$ over a right noetherian ring R of finite right Krull dimension is determined. Also, some partial results are obtained for an iterated skew-Laurent extension $R[\theta_1^{\pm 1},\dots,\theta_u^{\pm 1}]$ over a right noetherian ring R of finite right Krull dimension. In particular, a criterion is derived that indicates when such an iterated skew-Laurent extension can achieve the maximum possible Krull dimension.

Introduction. This paper is concerned with the Krull dimension (in the sense of Rentschler and Gabriel) of a skew-Laurent extension $T = R[\theta_1^{\pm 1},\dots,\theta_u^{\pm 1}]$, where R is a commutative noetherian ring of finite Krull dimension, equipped with u commuting automorphisms $\sigma_1,\dots,\sigma_u$. The main theorem states that the Krull dimension of T equals the maximum of the values

$$\text{height}(P) + \text{aut.dim.}(P)$$

as P ranges over the prime ideals of R, where aut.dim.(P) is a nonnegative integer that measures the invariance of P relative to products of powers of the σ_i. This theorem is developed in Part C of the paper. Part A is concerned with the question of the Krull dimension of a skew-Laurent extension $T = R[\theta, \theta^{-1}]$ over a right noetherian ring of finite right Krull dimension n, equipped with a single automorphism σ. In this portion of the paper, the main result is that T has Krull dimension n unless there exists a simple right R-module A such that $A \otimes_R T$ is not simple (as a T-module) and A has height n in the sense that there exist critical right R-modules $A = A_0, A_1,\dots,A_n$ such that each $A_i \otimes_R T$ is a critical T-module, each A_i is a minor subfactor of A_{i+1}, and A_n is a subfactor of R. If such an A does exist, then T has Krull dimension $n + 1$. This criterion is simplified when R is fully bounded. In this case it is shown that T has Krull dimension n unless R has a maximal two-sided ideal M of height n