PACIFIC JOURNAL OF MATHEMATICS
Vol. 114, No 2, 1984

AN ARTIN RELATION (MOD 2) FOR FINITE GROUP ACTIONS ON SPHERES

RONALD M. DOTZEL

Recently it has been shown that whenever a finite group G (not a p-group) acts on a homotopy sphere there is no general numerical relation which holds between the various formal dimensions of the fixed sets of p-subgroups (p dividing the order of G). However, if G is dihedral of order $2q$ (q an odd prime power) there is a numerical relation which holds (mod 2). In this paper, actions of groups G which are extensions of an odd order p-group by a cyclic 2-group are considered and a numerical relation (mod 2) is found to be satisfied (for such groups acting on spheres) between the various dimensions of fixed sets of certain subgroups; this relation generalises the classical Artin relation for dihedral actions on spheres.

0. Introduction. When a p-group P acts on a mod p homology n-sphere X, the fixed point set, X^H, of any subgroup H has the mod p homology of an $n(H)$-sphere, for some integer $n(H)$. The function from subgroups of P to integers defined by $H \to n(H)$ is called the dimension function and any such function arising in this way is known to originate in a real representation of P (see [**2**]). If P is elementary abelian, the Borel identity holds (see [**1**, pg. 175]):

$$n - n(P) = \sum (n(H) - n(P))$$

(sum over all $H \leq P$ such that $P/H = \mathbf{Z}_p$). The motivation for this identity comes from consideration of representations of P.

Now suppose G is the dihedral group D_p (p odd prime) (a semidirect product of $\mathbf{Z}_p$ and $\mathbf{Z}_2$ via the automorphism of $\mathbf{Z}_p$, $g \to g^{-1}$). If V is a real representation of G, one can by considering the real irreducible representations of G, write down the following Artin relation,

$$\dim V^G = \dim V^{\mathbf{Z}_2} - \left(\frac{\dim V - \dim V^{\mathbf{Z}_p}}{2}\right).$$

In [**3**], K. H. Dovermann and Ted Petrie show that for actions of D_p (and more generally any non p-group) on a homotopy sphere one cannot expect to find a numerical relation between the various dimensions of the fixed sets (in particular for smooth actions of D_p one cannot expect the Artin relation to hold). However, in [**8**, Thm. 1.3], E. Straume has shown that