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NONOSCILLATORY FUNCTIONAL DIFFERENTIAL EQUATIONS

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Our aim in this paper is to obtain sufficient conditions under which certain functional differential equations have a "large" number of nonoscillatory solutions. Using the characteristic equation of a "majorant" delay differential equation with constant coefficients and Schauder's fixed point theorem, we obtain conditions under which the functional differential equation in question has a nonoscillatory solution. Then a known comparison theorem is employed as a tool to demonstrate that if the functional differential equation has a nonoscillatory solution, then it really has a "large" number of such solutions.

Our aim in this paper is to obtain sufficient conditions under which the functional differential equation

$$x'(t) + \sum_{i=1}^{n} p_i(t)x(t - \tau_i(t)) = 0 \tag{1}$$

has a "large" number of nonoscillatory solutions. It is to be noted that the literature is scarce concerning conditions under which there exist nonoscillatory solutions. Using the characteristic equation of a "majorant" delay differential equation with constant coefficients and Schauder's fixed point theorem, we obtain conditions under which (1) has a nonoscillatory solution. Then we employ a known comparison theorem [see **1**, p. 224, also **4**, Ch. 6] as a tool to demonstrate that if (1) has a nonoscillatory solution then it really has a "large" number of such solutions.

As it is customary, a solution is said to be oscillatory if it has arbitrarily large zeros. A differential equation is called oscillatory if all of its solutions oscillate; otherwise, it is called nonoscillatory. In this paper we restrict our attention to real valued solutions $x(t)$.

2. Non-oscillations.

THEOREM 1. *Consider the differential equation*

$$x'(t) + \sum_{i=1}^{n} p_i(t)x(t - \tau_i(t)) = 0 \tag{1}$$