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# WEIGHTED REVERSE WEAK TYPE INEQUALITIES FOR THE HARDY-LITTLEWOOD MAXIMAL FUNCTION

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**Kenneth Andersen and Wo-Sang Young recently obtained a sufficient condition and a different necessary condition on a pair of weight functions for which a reverse weak type norm inequality holds for the Hardy-Littlewood maximal function. It is shown here that their necessary condition is also sufficient. The consequences of the sufficiency theorem that they proved are strengthened by use of this result.**

**1. Introduction.** Define the oriented cubes in $R^n$ to be those with sides parallel to the coordinate axes, and for a set $E$ in $R^n$ let $|E|$ denote the Lebesgue measure of $E$. For $f$ locally integrable on $R^n$, define the Hardy-Littlewood maximal function $Mf$ by

$$(Mf)(x) = \sup \frac{1}{|Q|} \int_Q |f(t)|\,dt,$$

where the supremum is taken over all oriented cubes containing $x$. The local sufficiency result proved in [1] for a reverse weak type inequality for the Hardy-Littlewood maximal function is as follows.

THEOREM (1.1). *If $U(x)$ and $V(x)$ are nonnegative functions defined on an oriented cube $Q_0$ such that*

$$\frac{1}{|Q|} \int_Q U(x)\,dx \geq A \operatorname*{ess\,sup}_{x \in Q} V(x) \tag{1.2}$$

*for all oriented cubes $Q \subset Q_0$, then*

$$\int_{Q_0 \cap \{x:\, (Mf)(x) > \lambda\}} U(x)\,dx \geq \frac{A 2^{-n}}{\lambda} \int_{\{x:\, |f(x)| > \lambda\}} |f(x)| V(x)\,dx$$

*for $f$ supported on $Q_0$ and $\lambda \geq (1/|Q_0|) \int_{Q_0} |f(x)|\,dx$.*

The local necessity result proved in [1] is the following; note that for a cube $Q$ in $R^n$ the notation $SQ$ denotes the cube concentric with $Q$ with sides $S$ times as long.