PACIFIC JOURNAL OF MATHEMATICS
Vol 117, No 2, 1985

DETERMINING INCOMPRESSIBILITY OF SURFACES IN ALTERNATING KNOT AND LINK COMPLEMENTS

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In this paper we carry out the next necessary step in the study of closed incompressible surfaces in alternating link complements — determining when a given punctured surface in an alternating link complement is incompressible, pairwise incompressible and understanding when a "peripheral tubing" operation (which will produce a closed surface) preserves the incompressibility of a surface.

1. Introduction. Let $L \subset \mathbf{R}^3 \subset S^3 = \mathbf{R}^3 \cup \{\infty\}$ be a non-split prime link which is alternating with respect to the projection π: $\mathbf{R}^3 \to \mathbf{R}^2$. In [**M**] we introduced the notion of a standard position embedding with respect to $\pi(L)$ for incompressible, pairwise incompressible surfaces in $S^3 - L$. From Theorem 3(c) of [**M**] we can conclude that there are only finitely many such n-punctured surfaces for $n > 0$. However, for a given surface $S \subset S^3 - L$ in standard position, no method was given in [**M**] for determining whether S is incompressible or pairwise incompressible. Furthermore, understanding when a "peripheral tubing" operation preserves the incompressibility of a standard position surface was not discussed in [**M**].

In this paper we give a method for determining when a surface in standard position is incompressible and pairwise incompressible. Using results on branched surfaces in [**F-O**], we adapt our new method to understanding when a "peripheral tubing" operation preserves the incompressibility. Throughout this paper we use the same notation and terminology as in [**M**].

In §2 we amplify on the notion of standard position.

In §3 we develop the notion of a compressing disk or pairwise compressing disk in standard position with respect to a standard positioned surface. From an analysis of a disk in standard position we produce our sought after method. In particular, the following sufficient condition for incompressibility becomes evident.

THEOREM 1. *Let L be a non-split prime alternating link and suppose $S \subset S^3 - L$ is a surface in standard position. For S to be incompressible,*