PACIFIC JOURNAL OF MATHEMATICS
Vol 117, No 2, 1985

# ORTHOGONAL PRIMITIVE IDEMPOTENTS AND BANACH ALGEBRAS ISOMORPHIC WITH $l_2$

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**In this paper, a study of orthogonal primitive idempotents and minimal ideals in topological algebras with orthogonal bases has been made. Among other results, a structure theorem for Banach algebras with orthogonal bases has been proved, similar to Ambrose's structure theorem for $H^*$-algebras in the separable case. Furthermore, a necessary and sufficient condition is given for Banach algebras with orthogonal bases to be isomorphically homeomorphic with the Hilbert algebra $l_2$.**

**1. Introduction.** In our papers [5], [6], [7], we introduced the notions of orthogonal and cyclic bases in topological algebras and studied a number of properties of such algebras. For instance, we proved necessary and sufficient conditions under which a topological algebra with an orthogonal basis is isomorphically homeomorphic with the Fréchet algebra $s$ of all complex sequences ([7], [6]). Similar characterization theorems were proved for the Banach algebra $l_1$ of all complex sequences with absolutely convergent series [3] and the Banach algebra $c_0$ of all complex null sequences [4].

Here we are concerned with a study of orthogonal primitive idempotents in topological algebras with orthogonal bases. As is well-known, the existence of idempotents in semisimple rings or algebras enables one to represent such rings or algebras as a direct sum of simple rings or algebras. Such a consideration for Hilbert algebras has led us to doubly orthogonal idempotents and to a structure theorem for such algebras. The structure theorem for Banach algebras with orthogonal bases is stronger than the similar result for $H^*$-algebras proved by Ambrose [1] but only in the separable case.

Specifically, we prove general results regarding orthogonal primitive idempotents in §3. For instance, we identify the maximal family of all orthogonal primitive idempotents. In §4, we prove that there are lots of closed minimal ideals (Theorem 4.4) in any topological algebra with an orthogonal basis. This leads us to a structure theorem for such algebras. Moreover, each Banach algebra, if it has an orthogonal basis, can be expressed as a countable direct sum of simple Banach subalgebras, each of which is isomorphically homeomorphic with the field of complex numbers