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NOT EVERY LODATO PROXIMITY IS COVERED

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In a recent paper Reed wrote, "In fact it may be that all Lodato proximities are covered. I was unable to find a counterexample". (Remark 1.10)

The purpose of this note is to show that, in general, Lodato proximities are not covered.

1. Preliminaries. A *closed filter* $\mathscr{F}$ on a topological space (X, c) is a proper filter (that is, a filter which does not contain the empty set) which has a base consisting of only closed sets. Maximal (with respect to set inclusion) closed filters are all called *ultraclosed filters*. For more information on the concept of ultraclosed filters see Thron [3].

Ultrafilters are maximal proper filters on a set and *grills* are exactly the unions of ultrafilters. For a detailed discussion on ultrafilters and grills, see Thron [2].

A *basic proximity* π on a set X is a symmetric binary relation on the power set $\mathscr{P}(X)$ of X satisfying the conditions:

$$(A, B \cup C) \in \pi \Leftrightarrow (A, B) \in \pi \quad \text{or } (A, C) \in \pi,$$
$$A \cap B \neq \varnothing \Rightarrow (A, B) \in \pi,$$
$$(A, \varnothing) \notin \pi, \quad \forall A \subset X.$$

The pair (X, π) is called a *basic proximity space* provided π is a basic proximity on X.

For a basic proximity π on X, we define

$$c_\pi(A) = \{x \in X: (\{x\}, A) \in \pi\} \quad \text{for all } A \subset X.$$

It is easily verified that c_π is a symmetric (Čech) closure operator. For a basic proximity π, c_π need not be a Kuratowski closure operator.

A basic proximity π on X is called a *Lodato proximity* if the following condition is saitsfied:

$$(c_\pi(A), c_\pi(B)) \in \pi \Rightarrow (A, B) \in \pi.$$

If π is a Lodato proximity on X then c_π is a Kuratowski closure operator on X and hence (X, c_π) is a topological space.

Let (X, π) be a basic proximity space and $\mathscr{G}$ be a grill on X. Then $\mathscr{G}$ is called a π-*clan* if

$$(A, B) \in \pi \quad \text{for all } A, B \text{ in } \mathscr{G}.$$