PACIFIC JOURNAL OF MATHEMATICS
Vol. 118, No. 1, 1985

ON THE LOCATION OF THE ZEROS OF CERTAIN COMPOSITE POLYNOMIALS

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Let $P(z)=\sum_{j=0}^{n} C(n,j)A_jz^j$ and $Q(z)=\sum_{j=0}^{m} C(m,j)B_jz^j$ be two polynomials of degree n and m, respectively, $m\le n$ ($C(n,j)=$ binomial coefficient). In this paper we study the relative location of the zeros of $P(z)$ and $Q(z)$ when the coefficients of these polynomials satisfy an apolar type relation and obtain some results. As an application of these results, we present certain generalizations of results of Walsh, Szegö, DeBruijn and Kakeya.

More recently [1] the author has shown that if

$$P(z)=\sum_{j=0}^{n} C(n,j)A_jz^j,\quad A_0A_n\neq 0, \tag{1}$$

and

$$Q(z)=\sum_{j=0}^{m} C(m,j)B_jz^j,\quad B_0B_m\neq 0, \tag{2}$$

are two polynomials of degree n and m, respectively, $m\le n$, such that

$$\begin{aligned} C(m,0)B_0A_n &- C(m,1)B_1A_{n-1}\\ &+\cdots+(-1)^m C(m,m)B_mA_{n-m}=0, \end{aligned} \tag{3}$$

then the following holds.

(a) If $Q(z)$ has all its zeros in $|z|\ge r$, then $P(z)$ has at least one zero in $|z|\ge r$.

(b) If $P(z)$ has all its zeros in $|z|\le r$, then $Q(z)$ has at least one zero in $|z|\le r$.

Here we first show that this result equally holds if the circle $|z|=r$ is replaced by a more general circle C with center at a point c and radius r. In fact, we prove

THEOREM 1. *If $P(z)$ is a polynomial of degree n defined by* (1) *and $Q(z)$ is a polynomial of degree m defined by* (2), *$m\le n$, end if there coefficients satisfy the relation* (3), *then the following holds.*

(i) *If $Q(z)$ has all its zeros in $|z-c|\ge r$, then $P(z)$ has at least one zero in $|z-c|\ge r$.*